

# Time Warping Intro & Applications In Underwater Acoustics

## 時間扭換簡介及其於水中聲學之應用

Tan, Tsu Wei 譚子偉助理教授  
Marine Science Dep., ROC Naval Academy



Work supported in part by  
Taiwan NSTC 111-2611-M-012-001  
Office of Naval Research, award No. N00014-20-WX01312 and N00014-21-WX01234  
National Science Foundation, Grant No. OCE1657430

## I. Why Time Warping

- What is Warping?
- Single receiver (broadband) v.s. hydrophone array (spatial diversity)

## II. What is **Time Warping**?

- Ideal Waveguide as theory based example
- Signal Processing & Mathematics: Time Frequency analysis
- Step-by-step to isolate and **retrieve individual normal mode and its modal dispersion curve** by a single receiver

## III. Applications of Time Warping in shallow water environment

- Active sources- geoacoustic inversion, source localization, seabed attenuation
- Passively obtained signals – Noise Cross Correlation Function

# Why Time Warping ?

## U.S. to authorize COVID-19 vaccines in coming days

By ShareAmerica - Dec 11, 2020



President Trump speaks during the Operation Warp Speed Vaccine Summit at the White House on December 8. (© Evan Vucci/AP Images)

Not logged in | Talk | Contributions | Create account | Log in

Article | Talk | Read | View source | View history | Search Wikipedia

## Operation Warp Speed

From Wikipedia, the free encyclopedia

**Operation Warp Speed (OWS)** was a [public-private partnership](#) initiated by the United States government to facilitate and accelerate the development, manufacturing, and distribution of [COVID-19 vaccines](#), [therapeutics](#), and [diagnostics](#).<sup>[1][2]</sup> The first news report of Operation Warp Speed was on April 29, 2020,<sup>[3][4][5]</sup> and the program was officially announced on May 15, 2020.<sup>[1]</sup> It was headed by [Moncef Slaoui](#) from May 2020 to January 2021 and by [David A. Kessler](#) from January to February 2021.<sup>[6]</sup> At the end of February 2021, Operation Warp Speed was transferred into the responsibilities of the [White House COVID-19 Response Team](#).<sup>[7]</sup>

The program promoted mass production of multiple vaccines, and different types of vaccine technologies, based on preliminary evidence, allowing for faster distribution if clinical trials confirm one of the vaccines is safe and effective. The plan anticipated that some of these vaccines will not prove safe or effective, making the program more costly than typical vaccine development, but potentially leading to the availability of a viable vaccine several months earlier than typical timelines.<sup>[8]</sup>

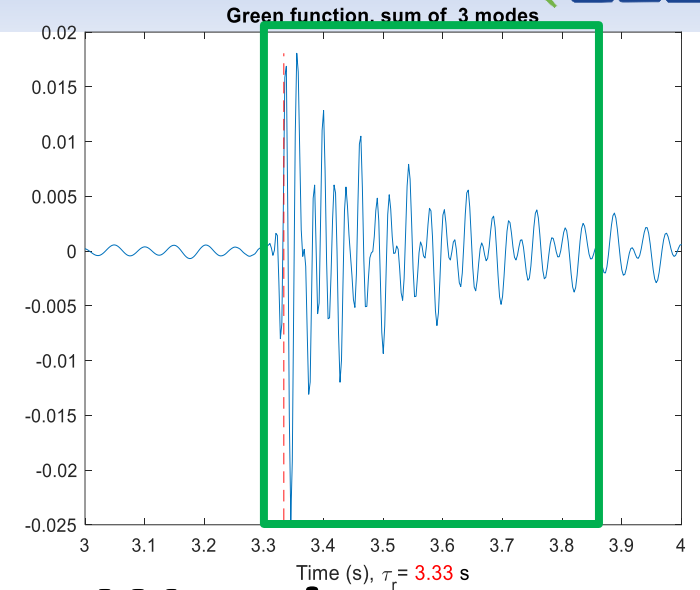
Operation Warp Speed, initially funded with about \$10 billion from the [CARES Act](#) (Coronavirus Aid, Relief, and Economic Security) passed by the [United States Congress](#) on March 27,<sup>[1]</sup> was an interagency program that includes components of the [Department of Health and Human Services](#), including the [Centers for Disease Control and Prevention](#), [Food and Drug Administration](#), the [National Institutes of Health](#), and the [Biomedical Advanced Research and](#)

Operation Warp Speed	
 Official seal of Operation Warp Speed	
<b>Active</b>	May 15, 2020 – February 24, 2021 (285 days)
<b>Disbanded</b>	Transitioned to <a href="#">White House COVID-19 Response Team</a>
<b>Country</b>	<a href="#">United States</a>
<b>Allegiance</b>	 <a href="#">United States</a>
<b>Part of</b>	<a href="#">U.S. Department of Defense</a> <a href="#">U.S. Department of Health and Human Services</a> Other various government agencies
<b>Engagements</b>	<a href="#">Coronavirus disease 2019</a>
<b>Website</b>	<a href="#">Coronavirus: Operation Warp Speed</a>

# Warping on Signals



Disney

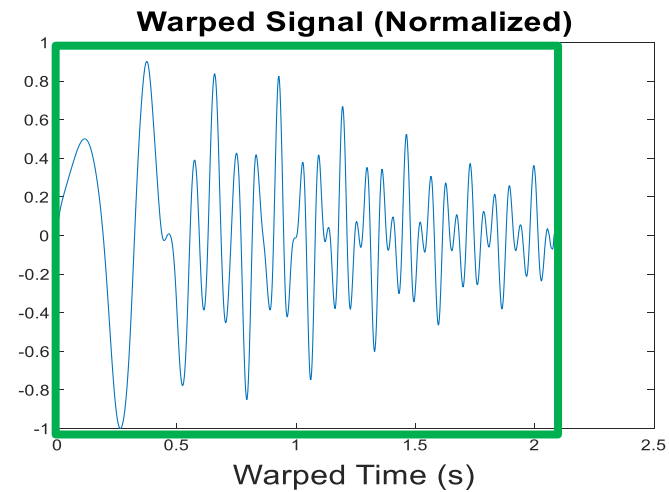


**Warping**



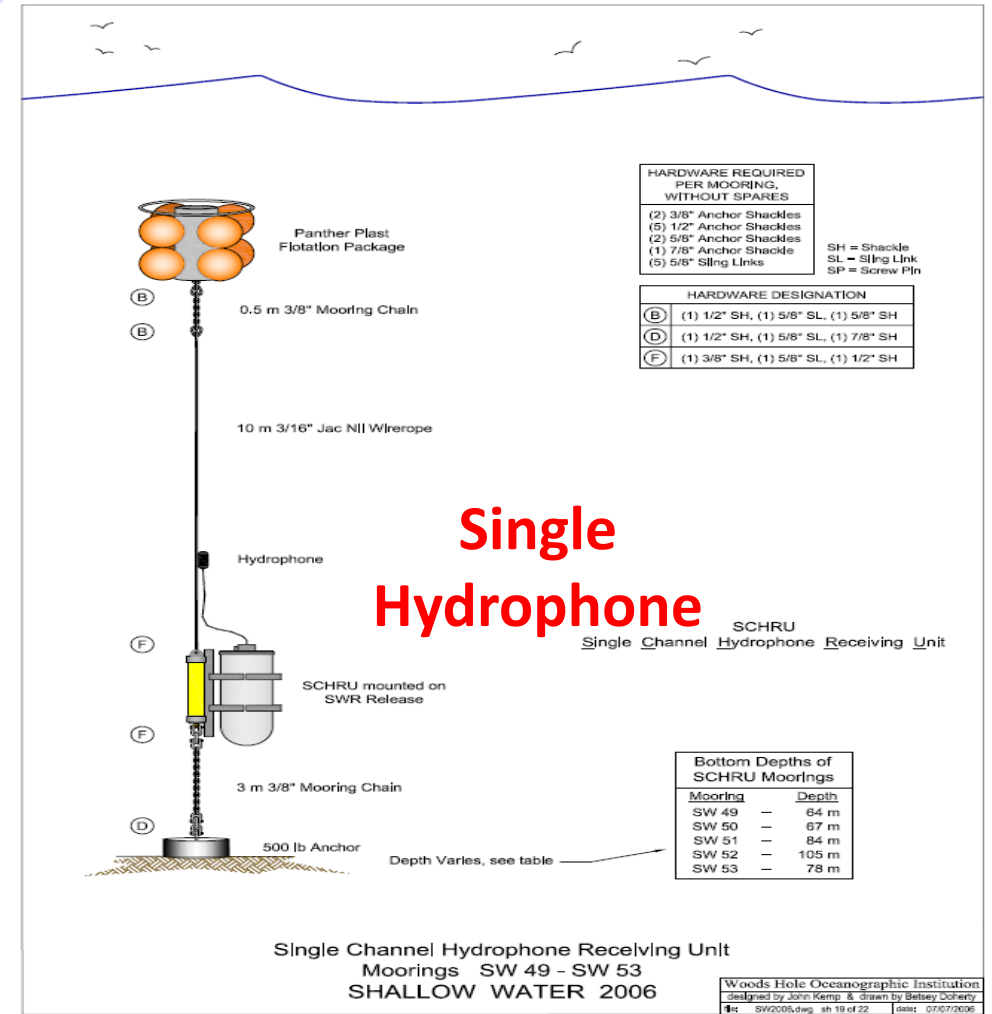
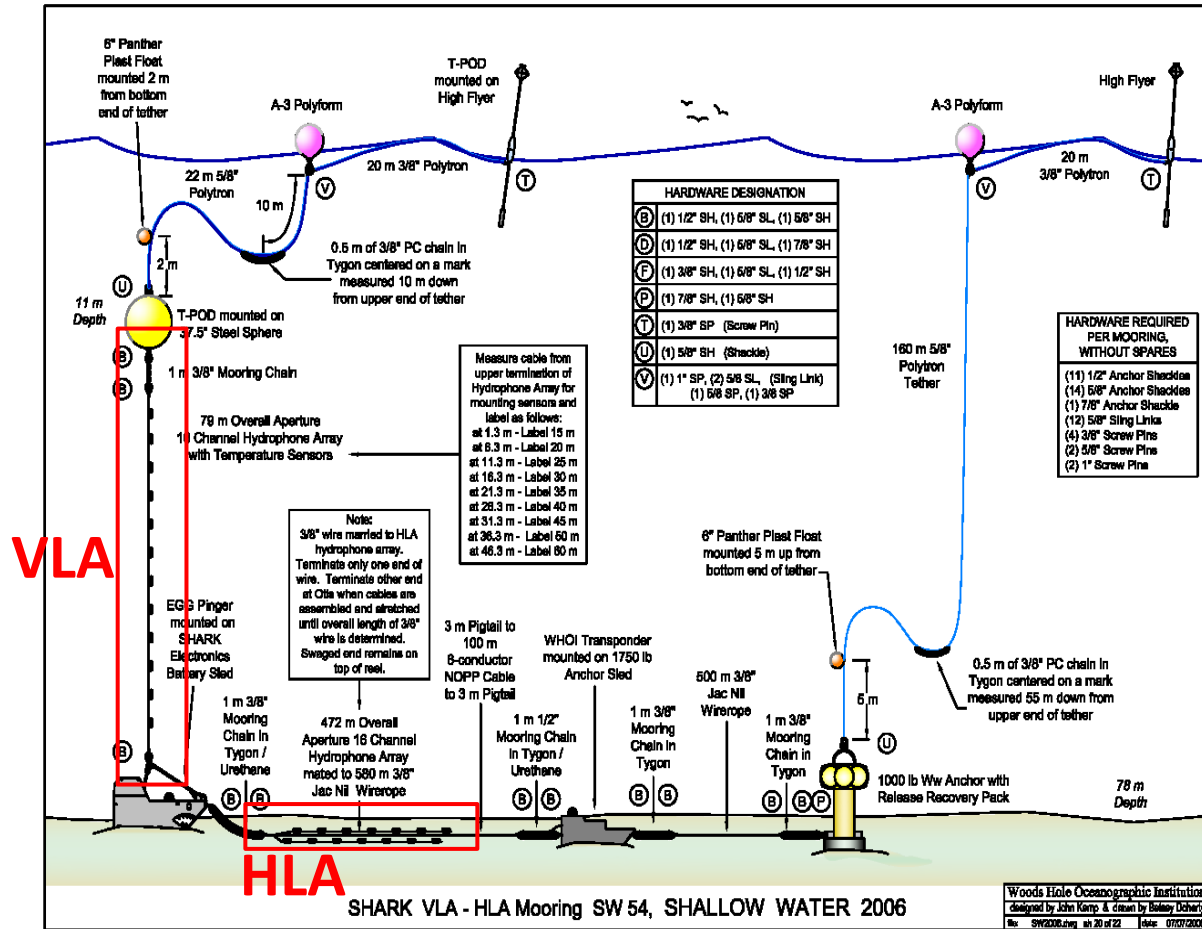
**Warping**

R. G. Baraniuk and D. L. Jones, "Unitary equivalence: A new twist on signal processing," *IEEE transactions on signal processing*, vol. 43, no. 10, pp. 2269-2282, 1995



# Why Time Warping in Underwater Acoustics?

Array (spatial diversity) VS Single Hydrophone (broadband frequency)



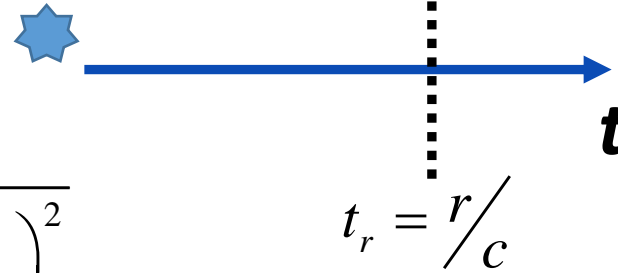
Arthur E. Newhall et al. "Acoustic and Oceanographic Observations and Configuration Information for the WHOI Moorings from the SW06 Experiment." May 2009 Technical Report

Phase

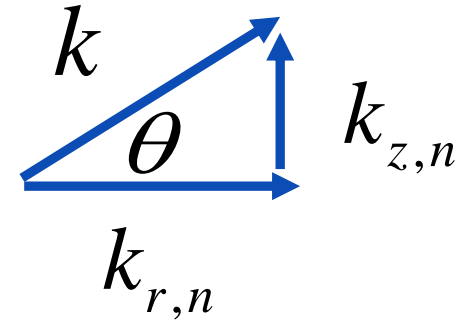
$$\phi_n(t) = \omega t - k_{r,n} r$$

sound speed =  $c$

Point Source



Received signal

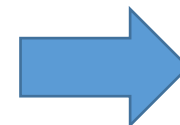


$$k_{r,n} = k \cos \theta = k \frac{\sqrt{k^2 - k_{z,n}^2}}{k} = k \sqrt{1 - \left(\frac{f_n}{f}\right)^2}$$

$$c_{g,n}(f) = c \cos \theta = c \sqrt{1 - \left(\frac{f_n}{f}\right)^2} = \frac{r}{t}$$

$$\omega t - k_{r,n} r = \frac{2\pi f_n}{\sqrt{1 - \left(\frac{r}{ct}\right)^2}} \left( t - \frac{r^2}{c^2 t} \right) = 2\pi f_n \sqrt{t^2 - t_r^2}$$

Linearize Phase in warped domain



$$w(t) = \sqrt{t^2 - t_r^2}$$

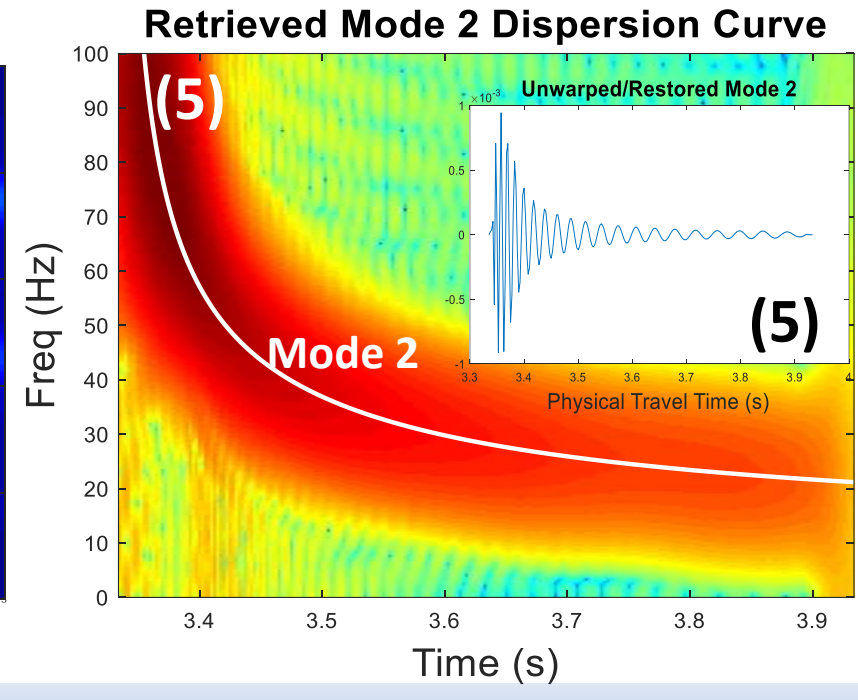
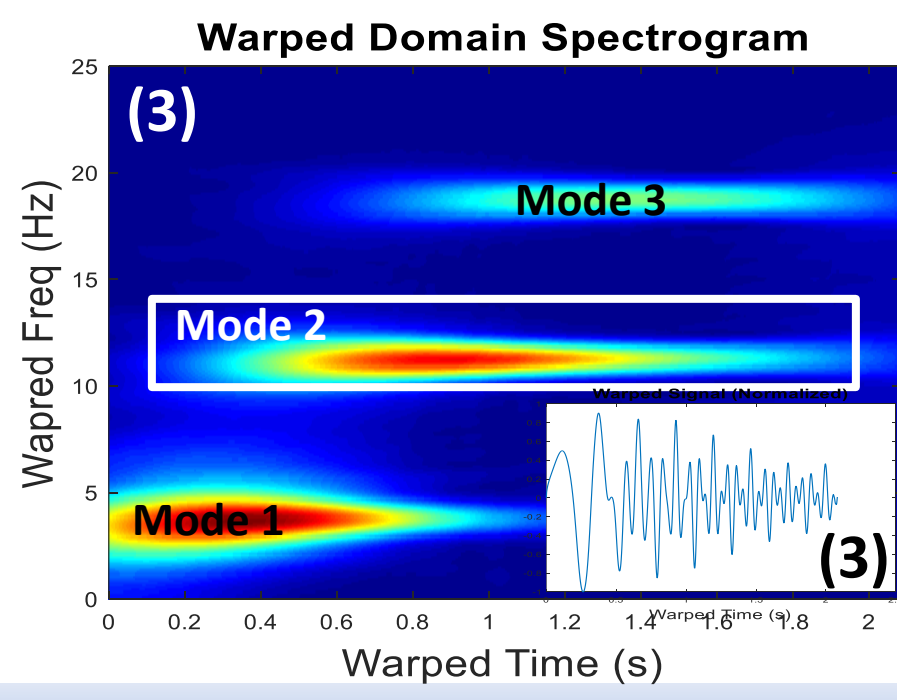
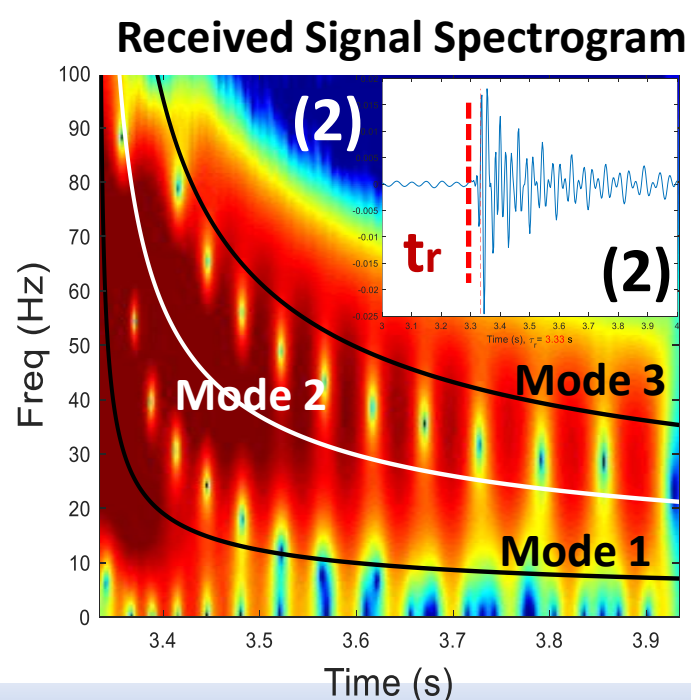
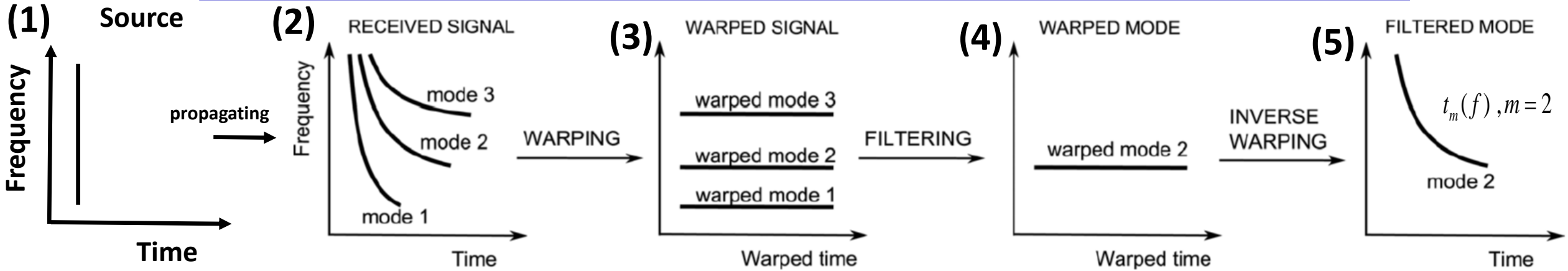


# Time Warping : Modal Filtering

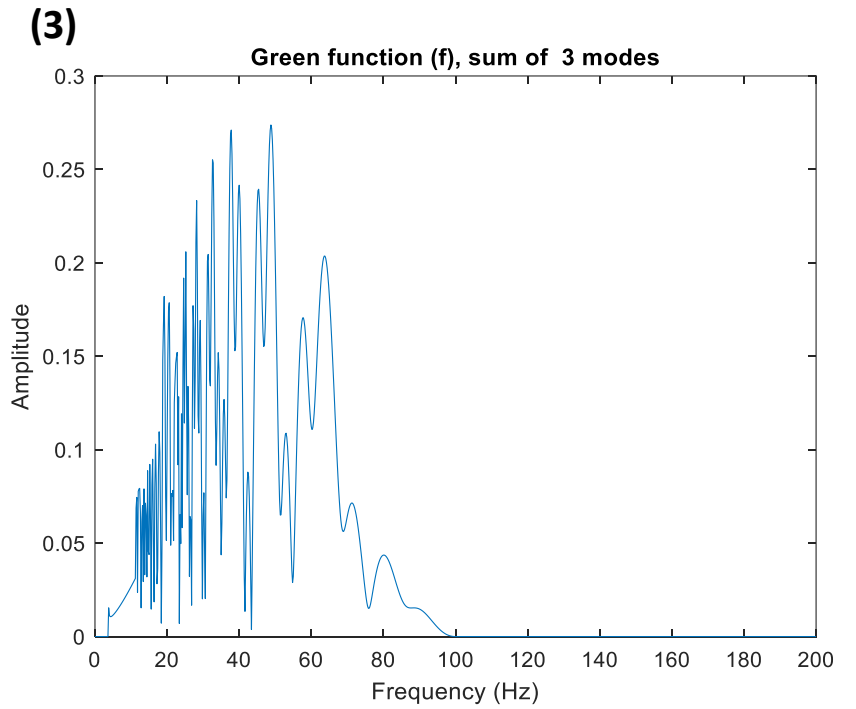
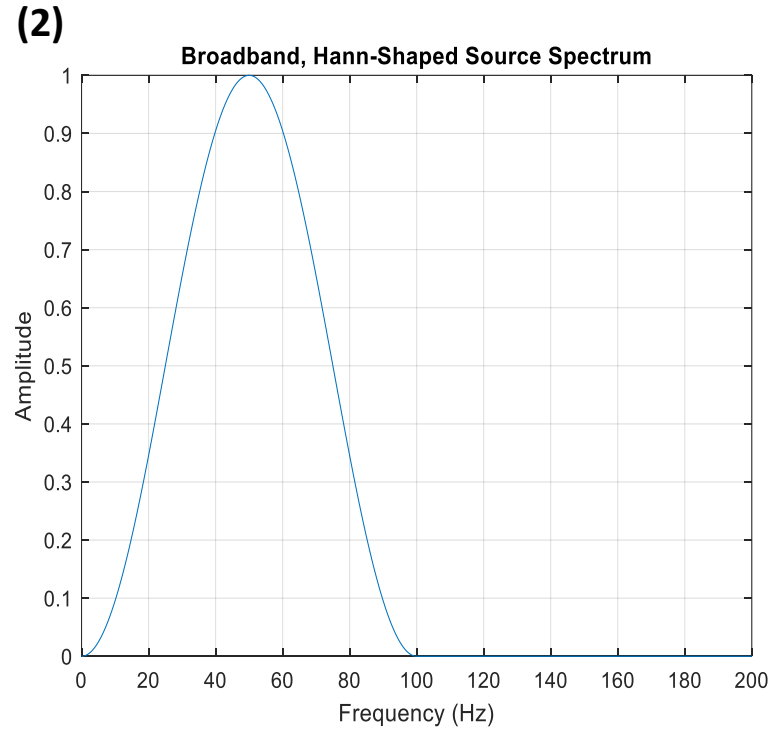
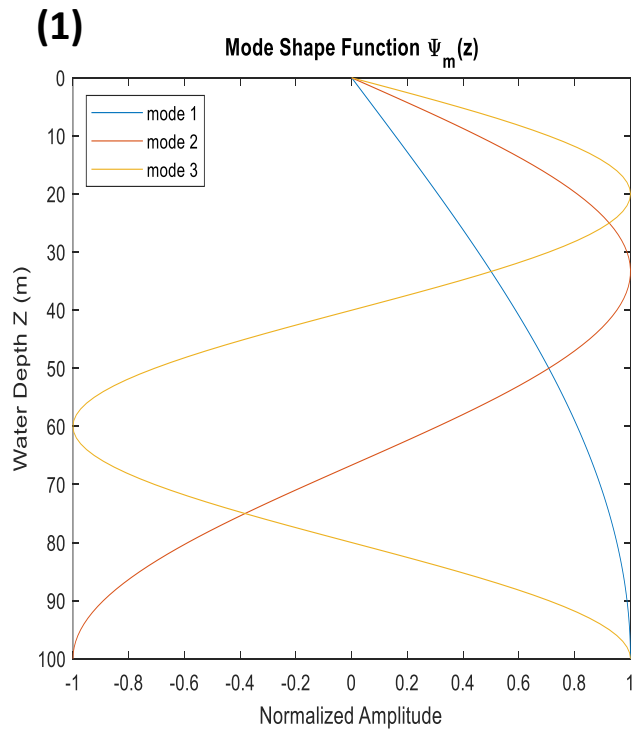


ROC Naval Academy

Julien Bonnel et al "Nonlinear time-warping made simple: A step-by-step tutorial on underwater acoustic modal separation with a single hydrophone" *J. Acoust. Soc. Am* 147,1897 (2020)







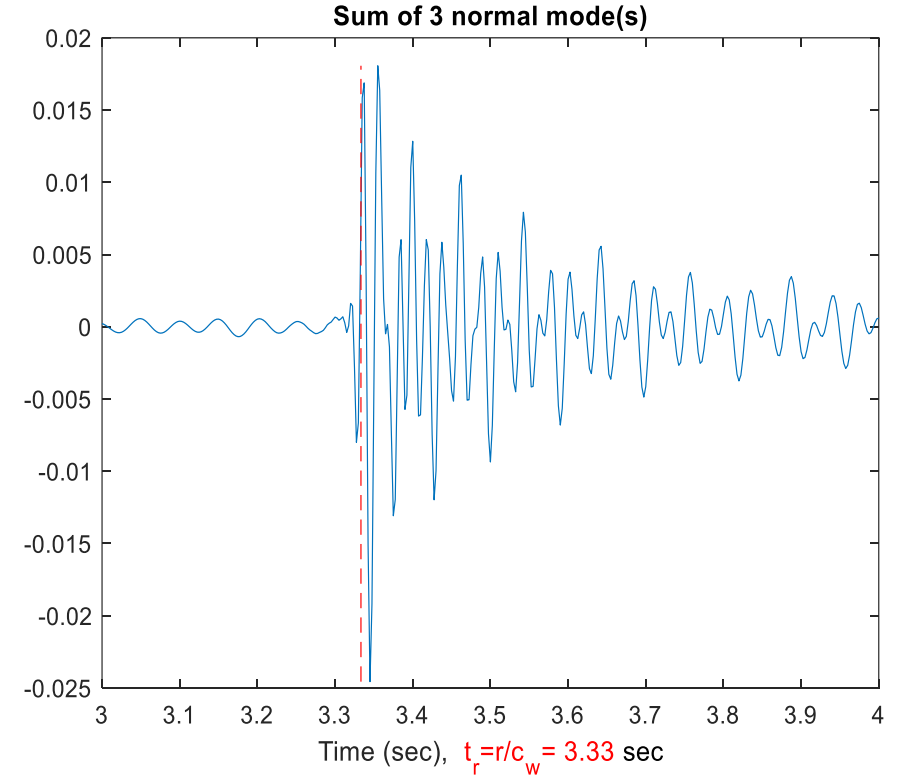
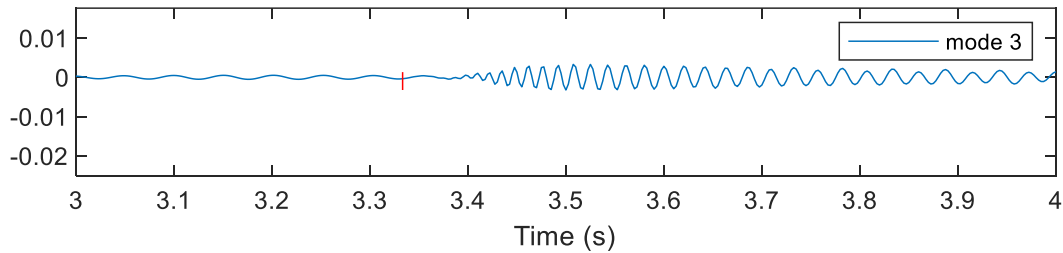
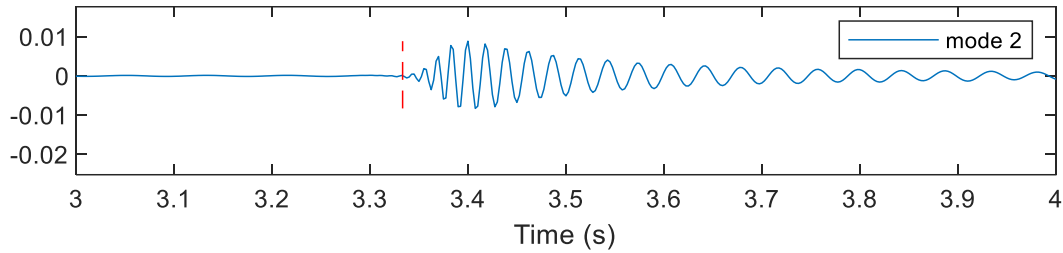
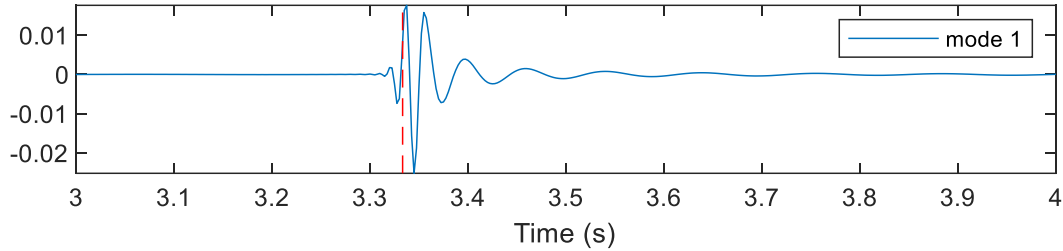
$$\psi_m(z) = \sqrt{\frac{2\rho_w}{H}} \sin\left(\frac{\pi z}{2H}(2m-1)\right)$$

$$G(z_1, z_2, r, \omega) = \frac{-i}{4} \sum_m \psi_m(z_{source}) \psi_m(z_{receiver}) H_0^{(1)}(\xi_m r)$$

$$\xi_m = \sqrt{\frac{\omega^2}{c_w^2} - \frac{\pi^2(2m-1)^2}{4H^2}}$$

$$H_0^{(1)}(\xi_m r) \approx \sqrt{\frac{2}{\pi \xi_m r}} \exp\left(i\xi_m r - \frac{i\pi}{4}\right)$$

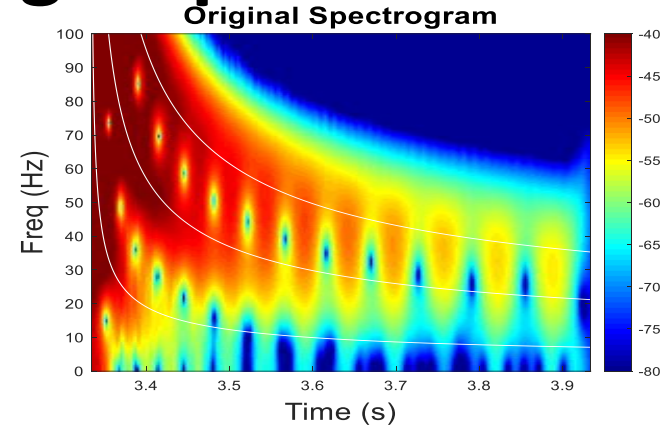
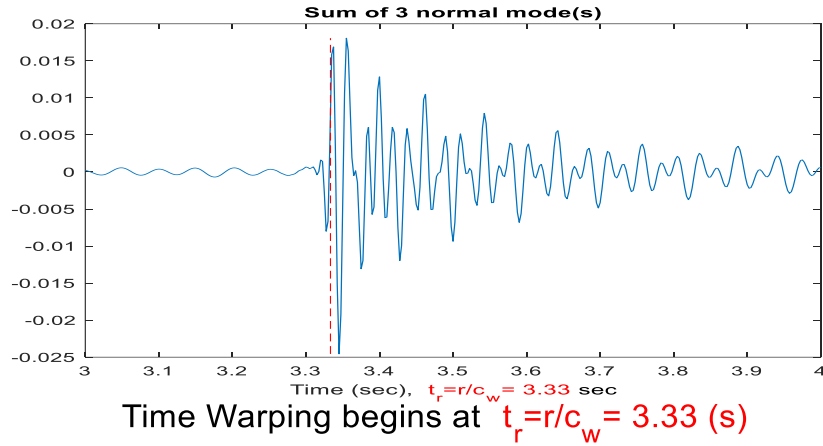
Time Warping begins at  $t_r = r/c_w = 3.33$  (s)



**Inverse FT on** 
$$G(z_1, z_2, r; \omega) = \frac{-i}{4} \sum_m \psi_m(z_{source}) \psi_m(z_{receiver}) H_0^{(1)}(\xi_m r)$$

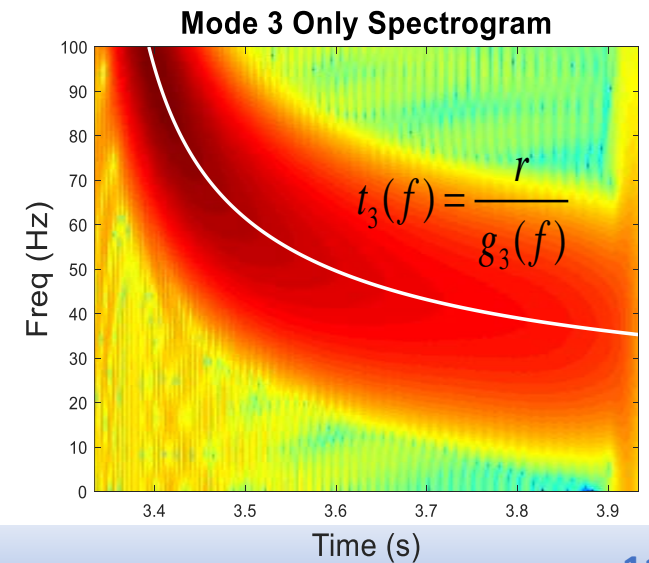
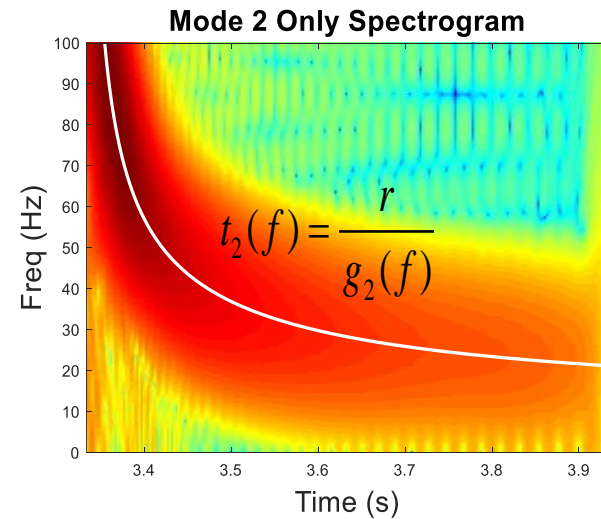
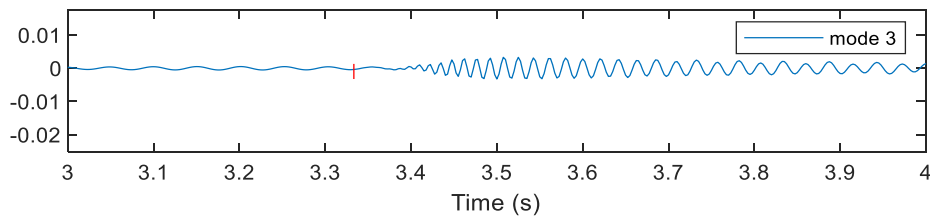
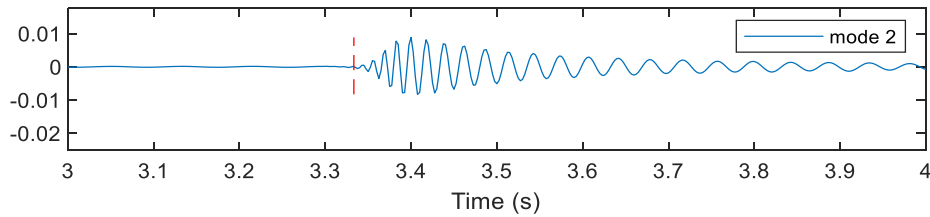
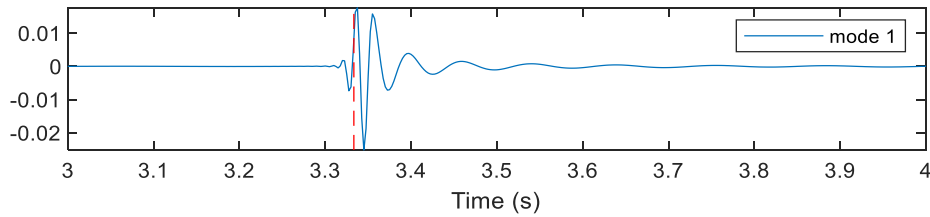
# Building Spectrogram

## ~~measuring dispersion curves~~



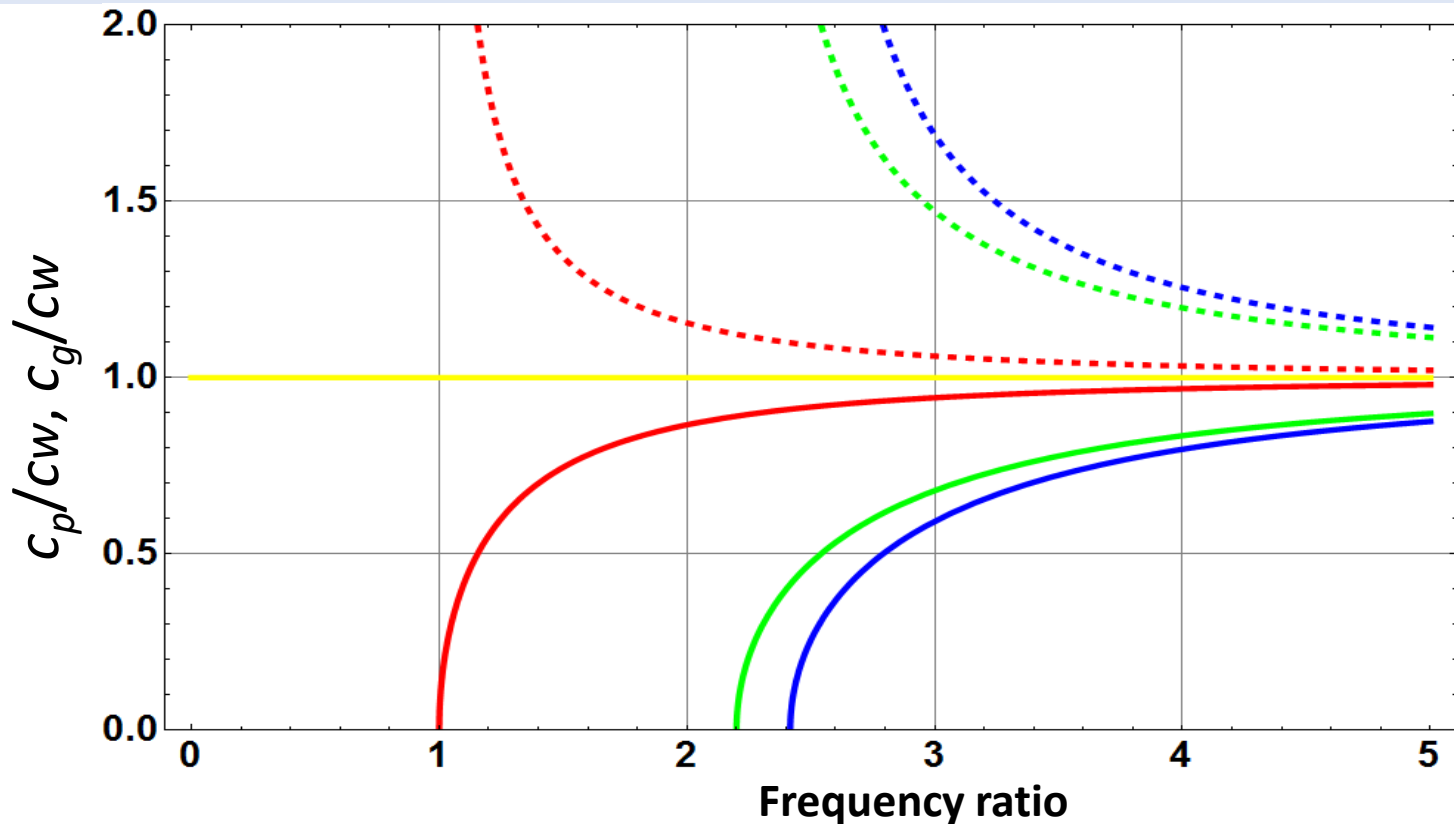
$$|\text{STFT}_Y(t, f)|^2$$

$$\text{STFT}_Y(t, f) = \int_{-\infty}^{\infty} y(\tau) h^*(\tau - t) e^{-i2\pi f \tau} d\tau$$



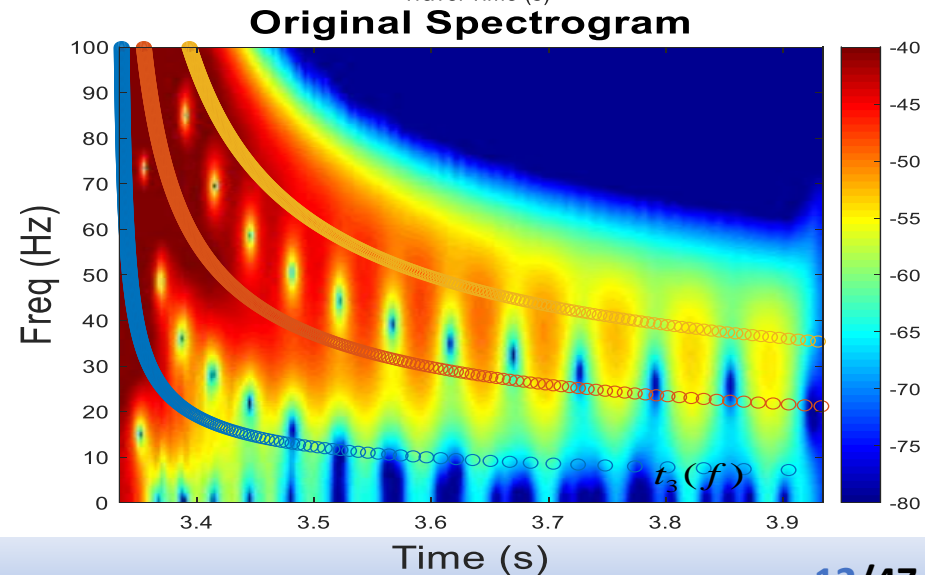
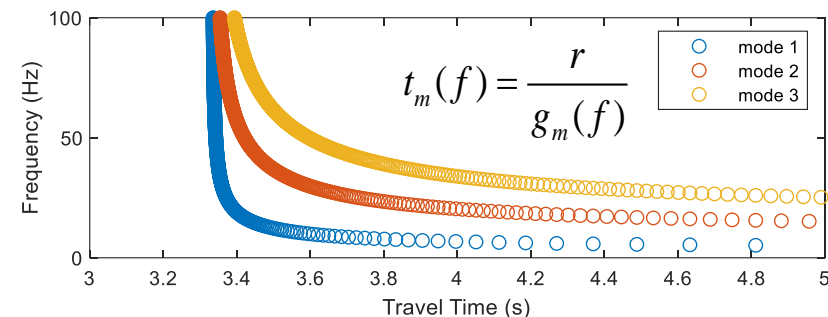
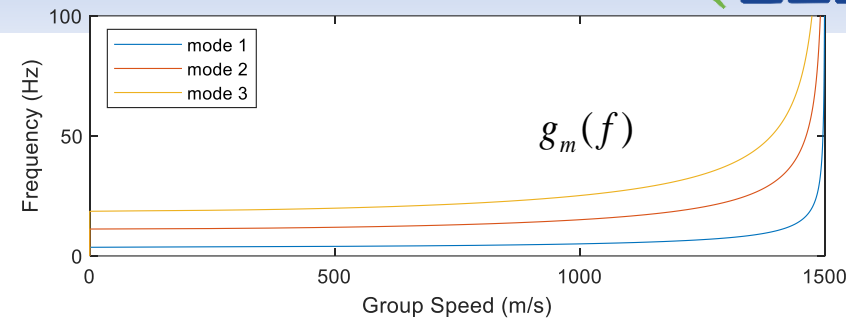


# Time(Group Speed)-Frequency Analysis Dispersion Curves

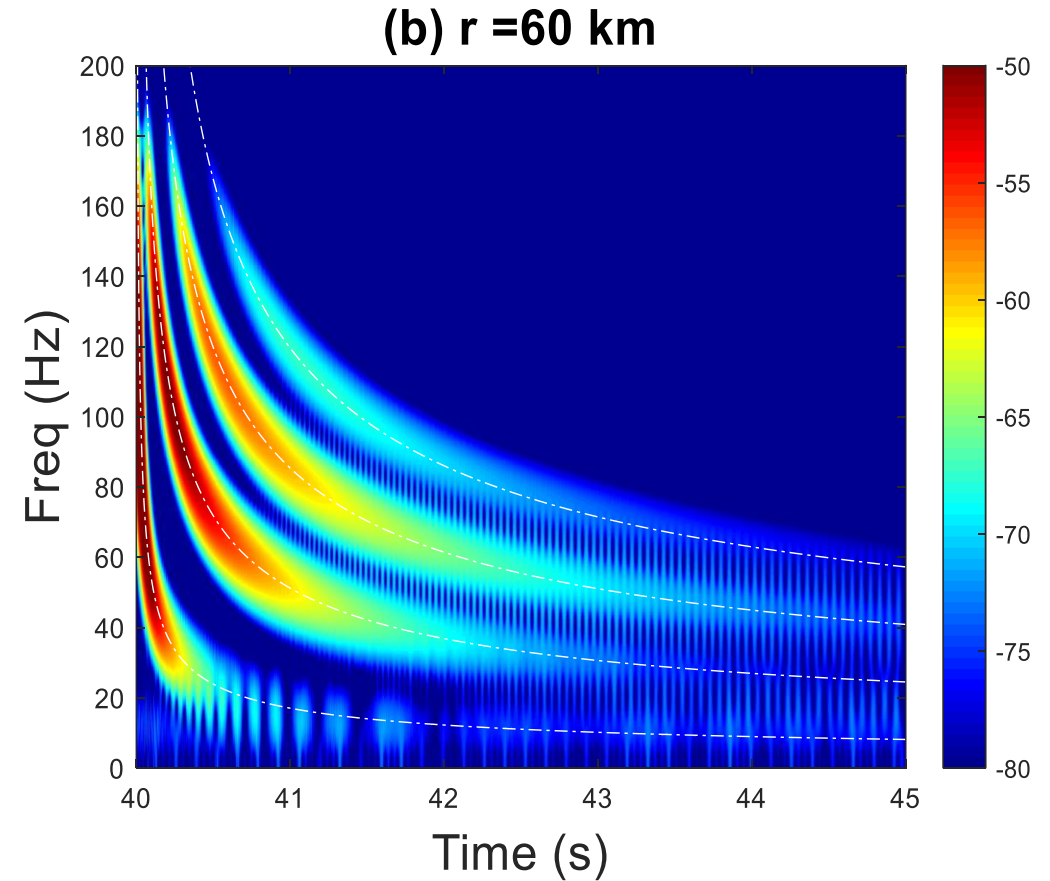
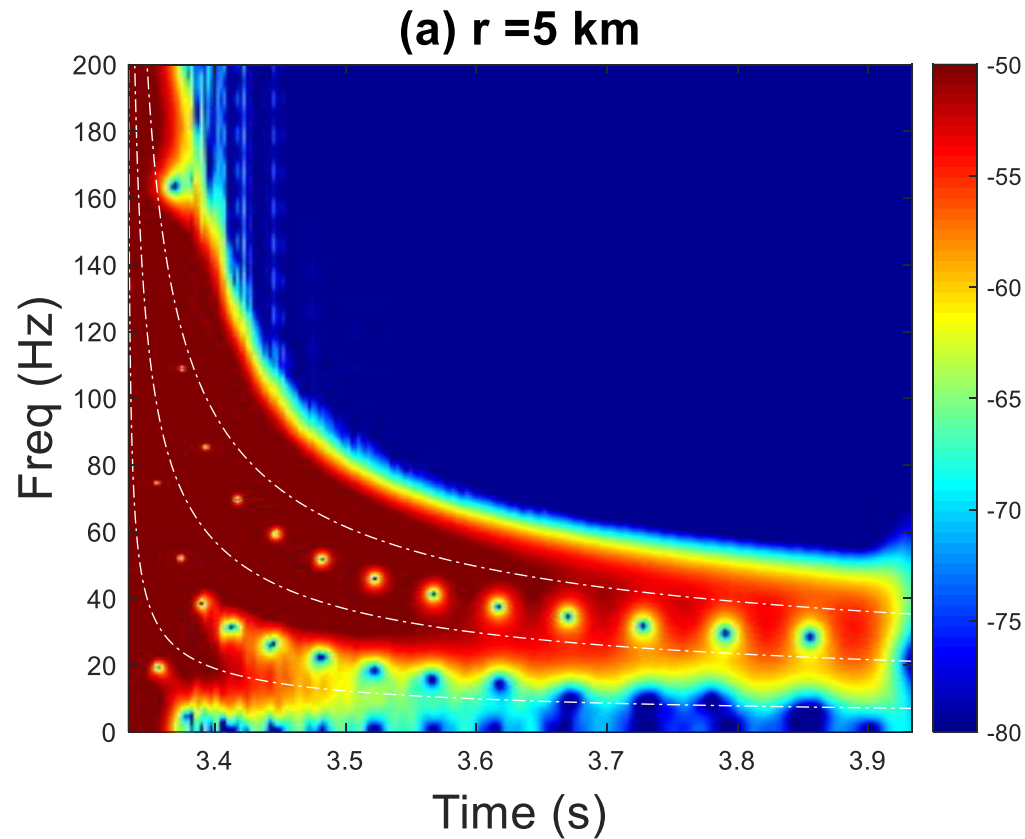


$$v_m(\omega) = \frac{\omega}{\xi_m} = c_w \left( 1 - \frac{\pi^2 (2n-1)^2 c_w^2}{4H^2 \omega^2} \right)^{-1/2}, \quad g_m(\omega) = \frac{\partial \omega}{\partial \xi_m} = c_w \left( 1 - \frac{\pi^2 (2n-1)^2 c_w^2}{4H^2 \omega^2} \right)^{1/2}$$

Group Speed  $g_m$  contains environmental information,  $H$ ,  $C_w$ , including seabed properties in real environment



# Modal Separation Restriction



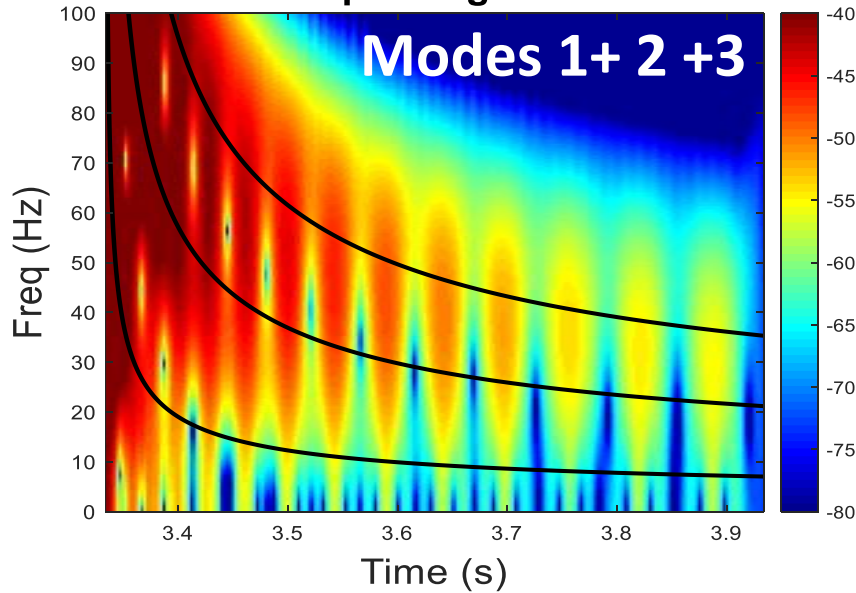
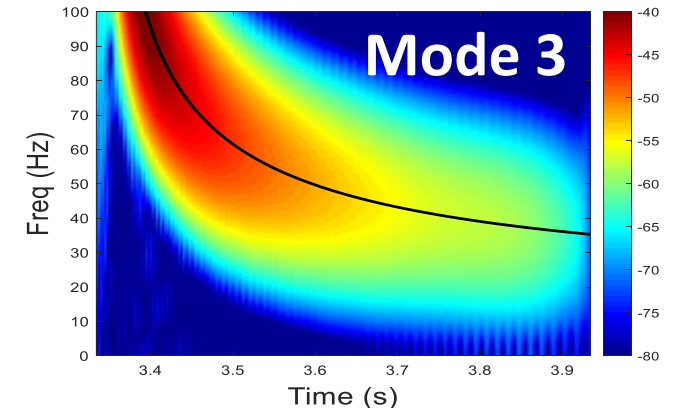
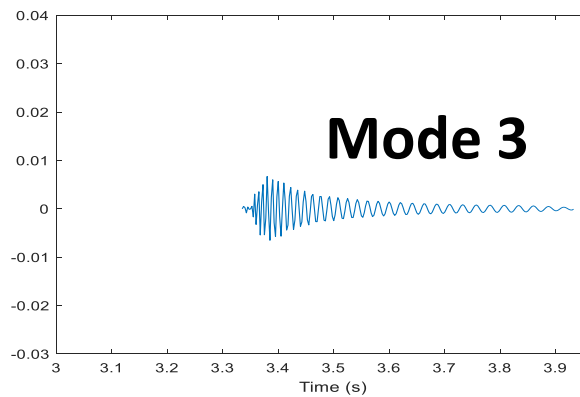
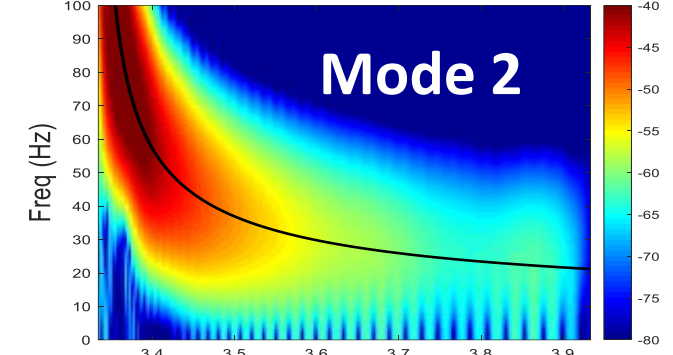
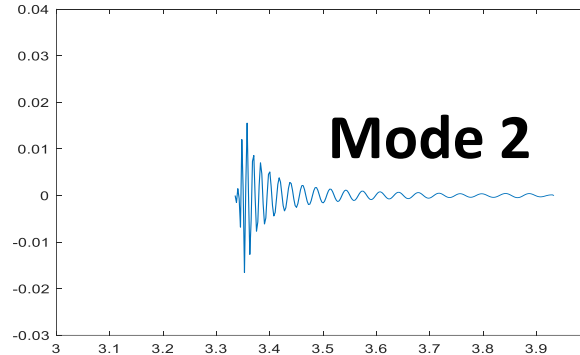
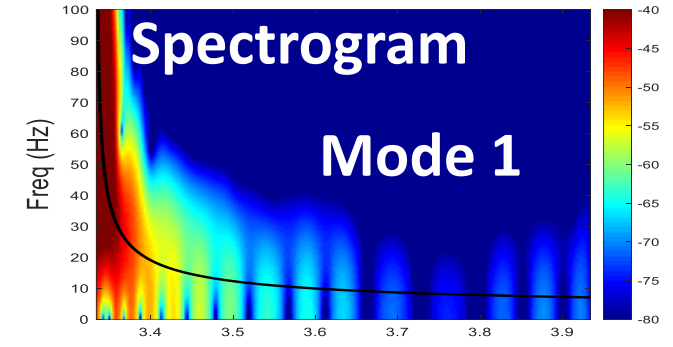
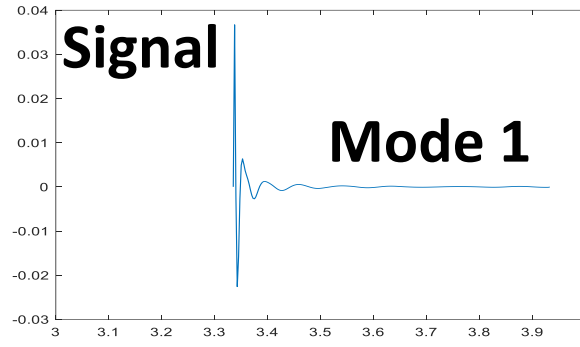
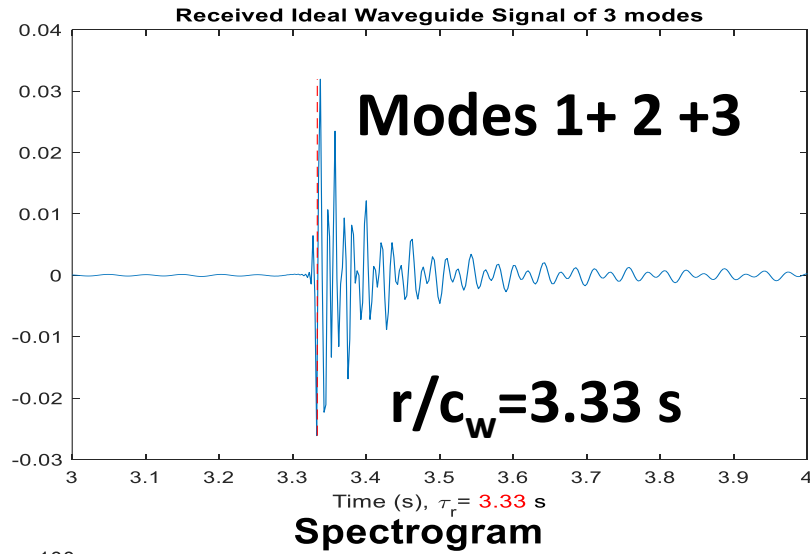
$$t_m(f) = \frac{r}{g_m(f)}$$

(1)  $\Delta t_{\min} > T_h$

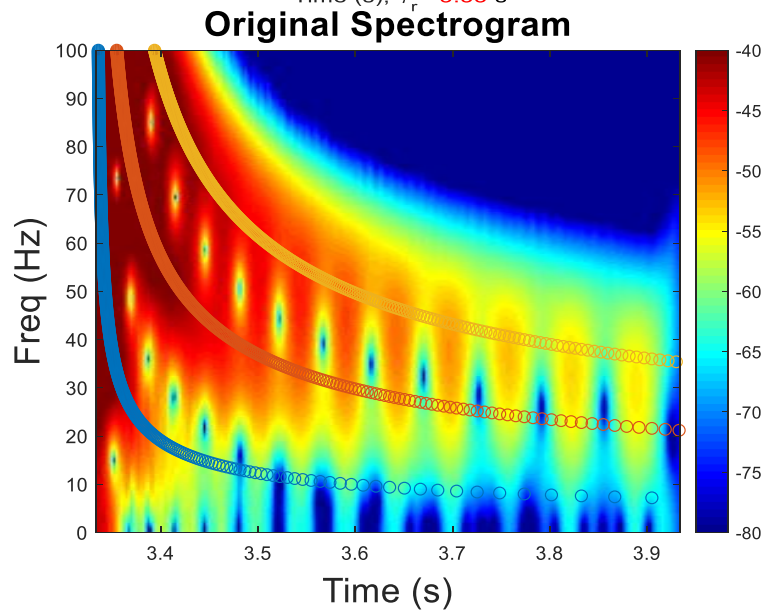
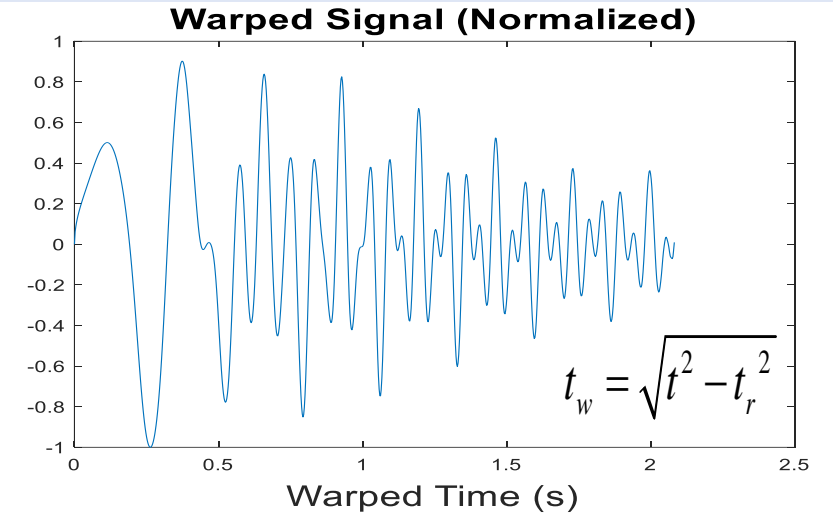
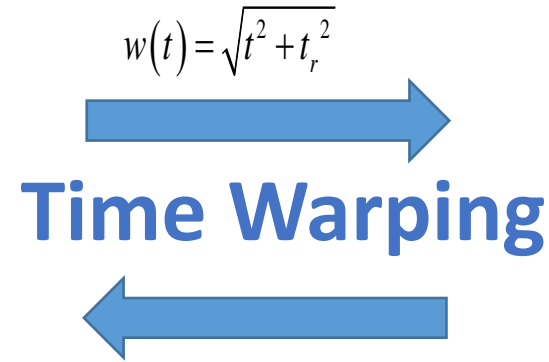
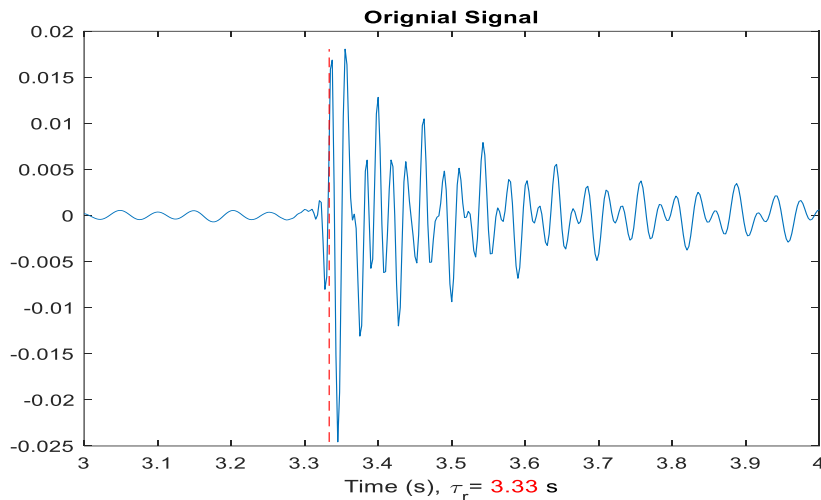
(2)  $\Delta f_{\min} > \alpha / T_h, \alpha = 4$

(3)  $\Delta t_{\min} > \frac{\alpha}{\Delta f_{\min}}, \alpha = 4$

# Objective of Time Warping



# Time Warping: Quick View

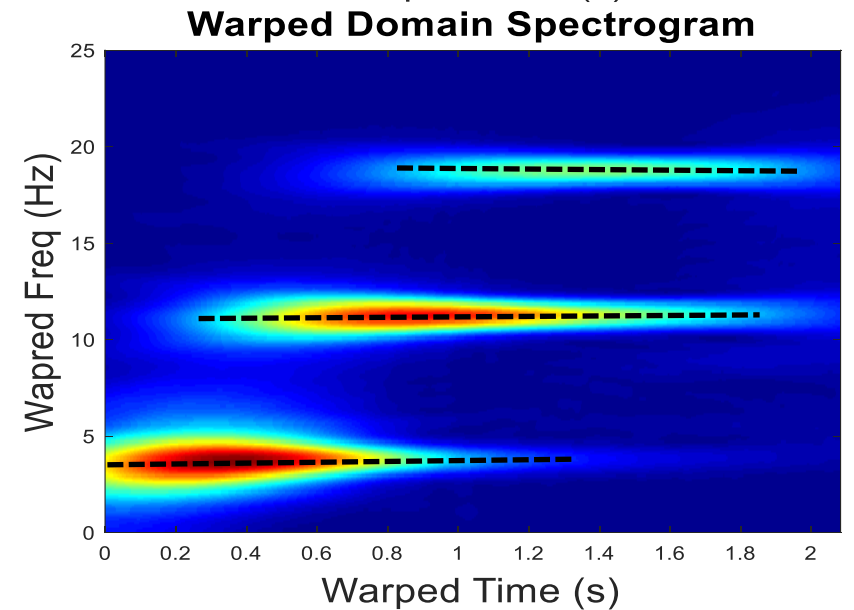


$$f_{w,m} = f_m(t) \sqrt{1 - \left(\frac{t_r}{t}\right)^2}$$

$$= F_m = \frac{2m-1}{4H} C_w$$

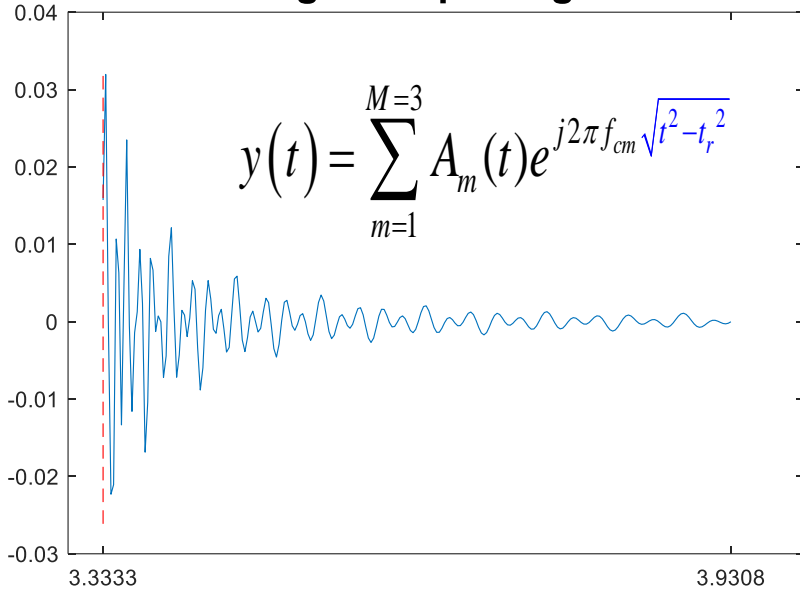
$$H = 100m; C_w = 1500m/s$$

$$F_1 = 3.75; F_2 = 12.5; F_3 = 18.75 \text{ Hz}$$



# II. Time Warping: Theory

Original Input Signal

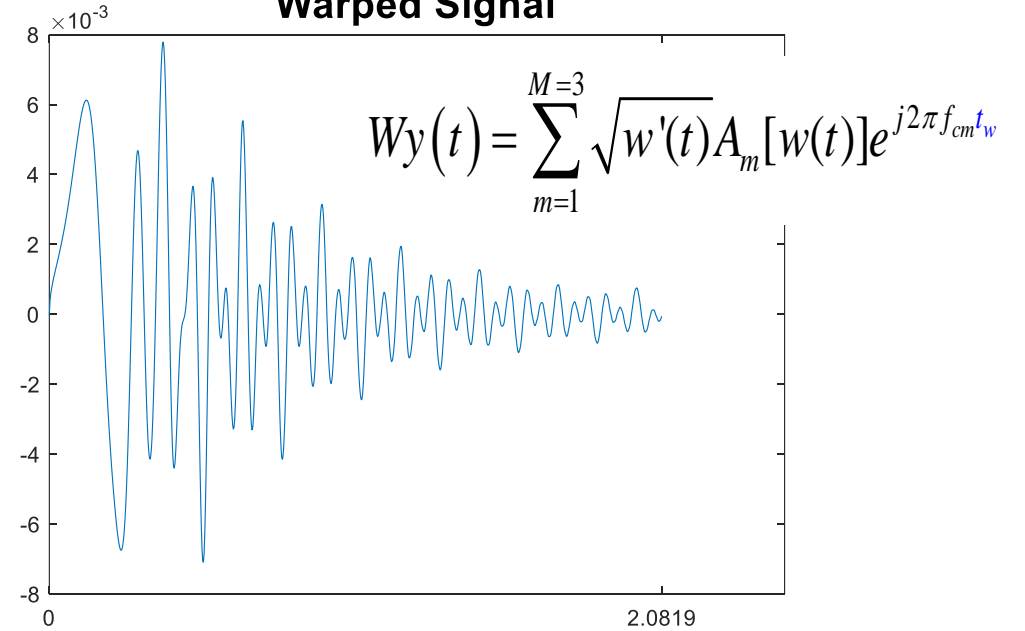


Time (s),  $t_r = 3.33$  s

$$t_w = \sqrt{t^2 - t_r^2}$$

$$Wy(t) = \sqrt{|w'(t)|} y[w(t)]$$

Warped Signal



Warped Time (s)

Warping Operator  $w(t) = \sqrt{t^2 + t_r^2}$

Unwarping Operator  $w^{-1}(t) = \sqrt{t^2 - t_r^2}$

$$\Phi_m(t) = \zeta_m(t) r \quad \Phi_m^{id}(t) = 2\pi f_{cm} \sqrt{t^2 - t_r^2}$$

$$t_r = \frac{r}{c_w} \quad \Phi_m^{id}(t) \rightarrow \Phi_m^{id}(t_w) = 2\pi f_{cm} t_w$$

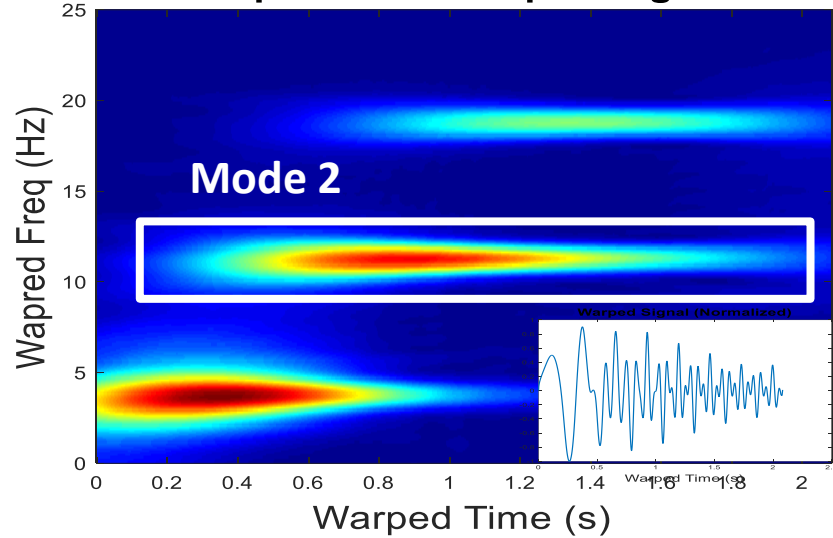
$$W^{-1}Wy(t) = WW^{-1}y(t) = y(t)$$

Julien Bonnel, Barbara Nicolas, Jerome I. Mars and Shane C. Walker "Estimation of modal group velocities with a single receiver for geoacoustic inversion in shallow water" *J. Acoust. Soc. Am* 128, 719-727(2010)

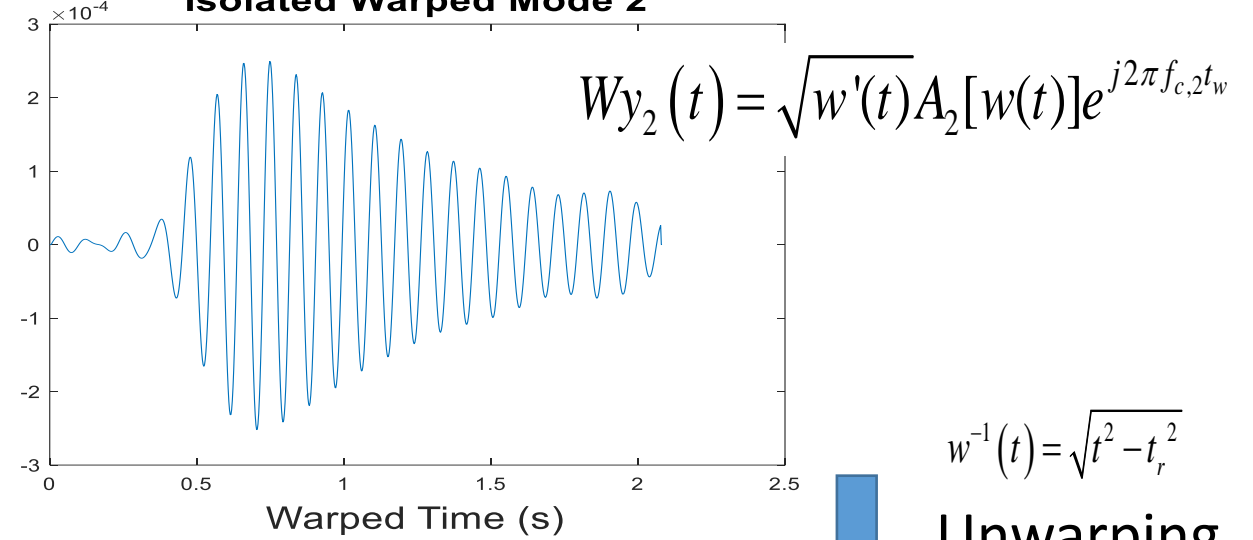


# II. Modal Separation by Time Warping

Warped Domain Spectrogram

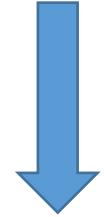


Isolated Warped Mode 2

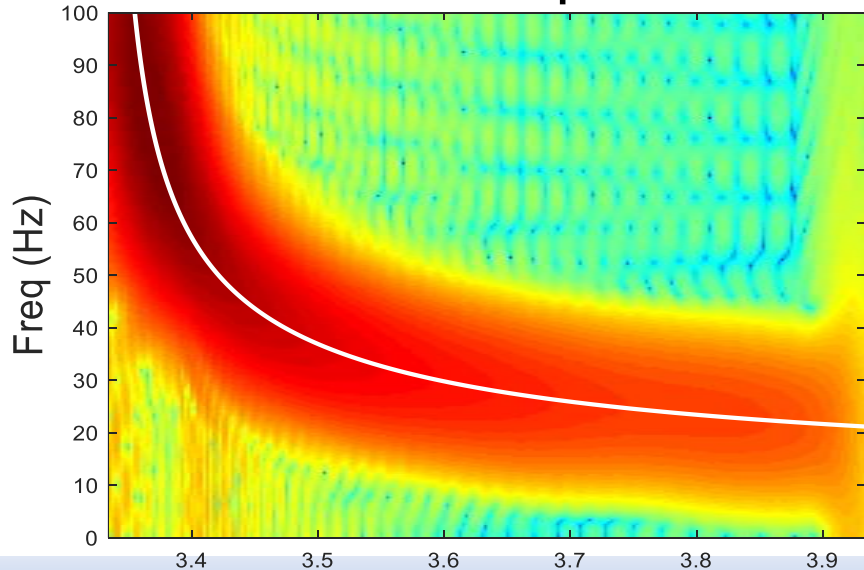


$$w^{-1}(t) = \sqrt{t^2 - t_r^2}$$

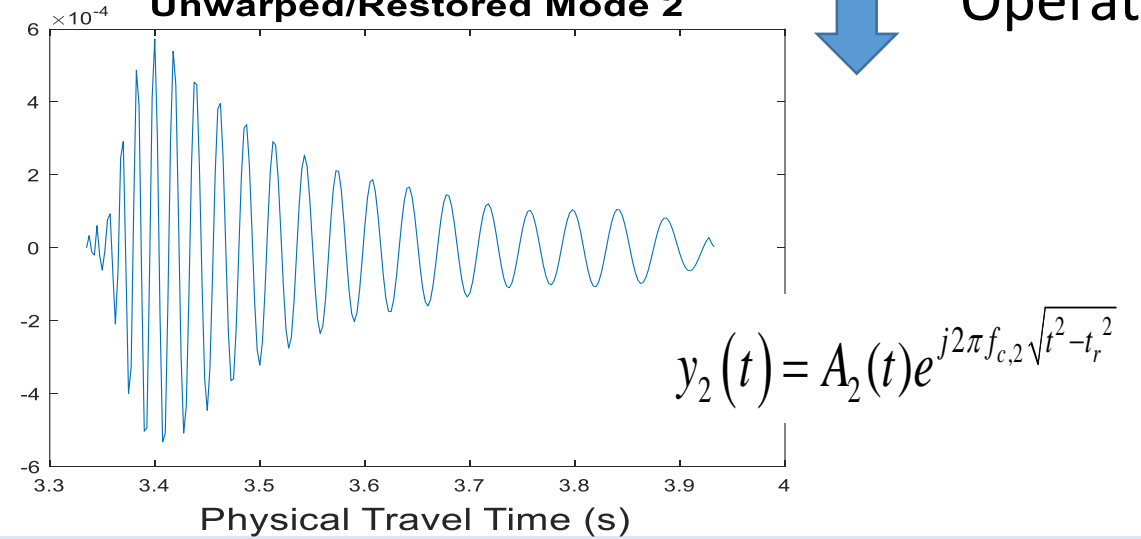
Unwarping Operator



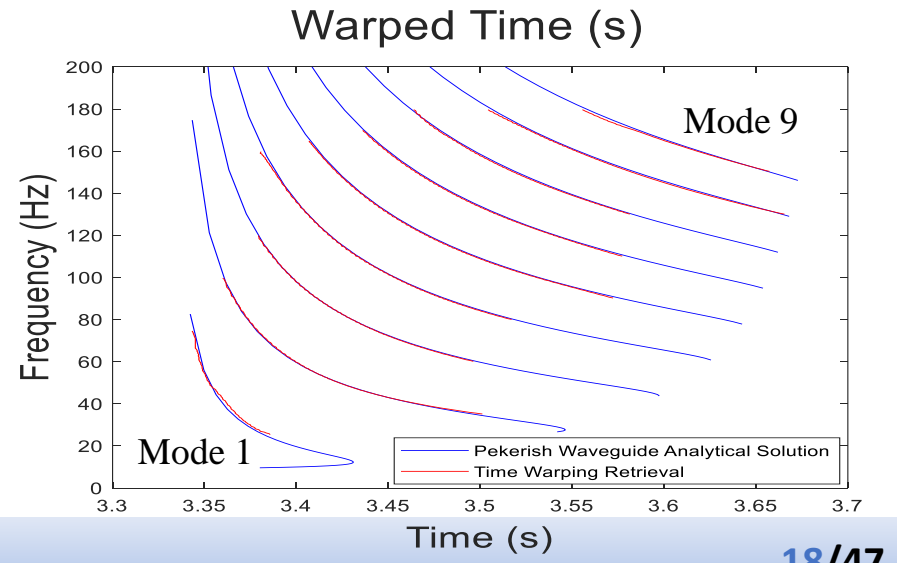
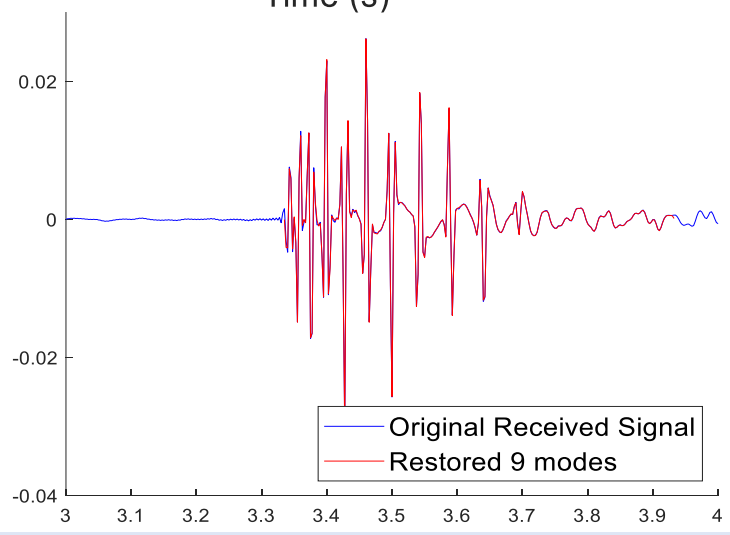
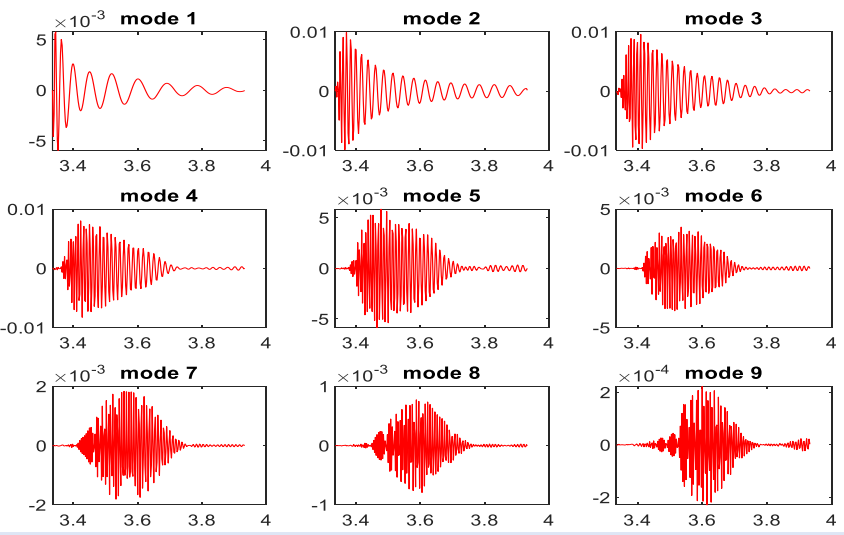
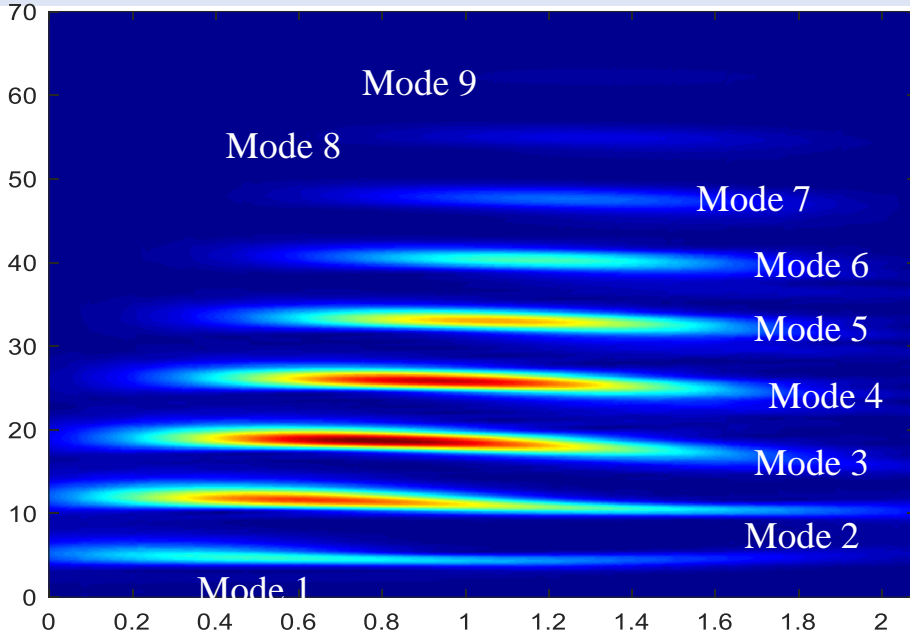
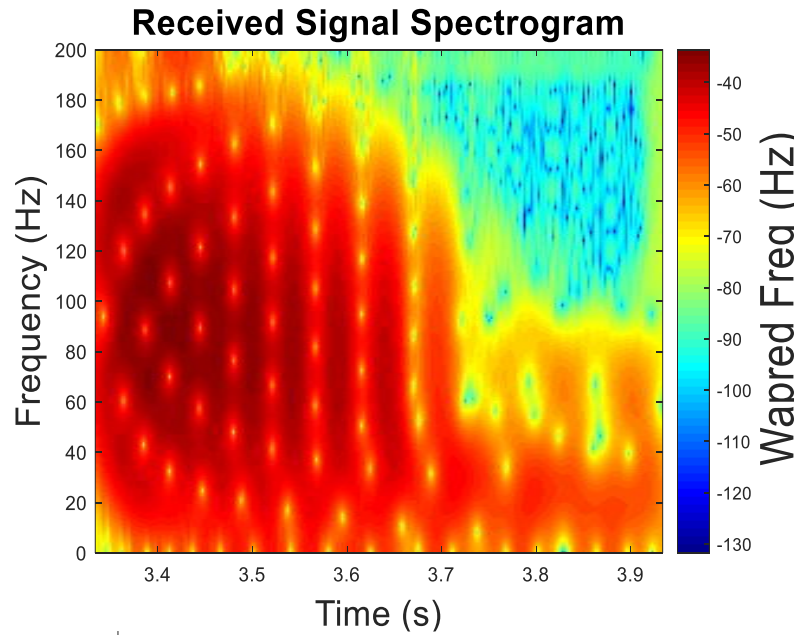
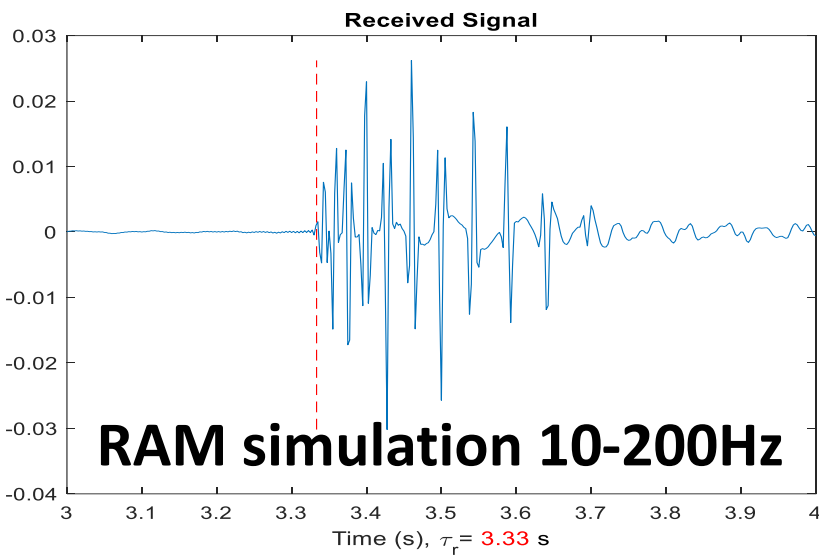
Retrieved Mode 2 Dispersion Curve



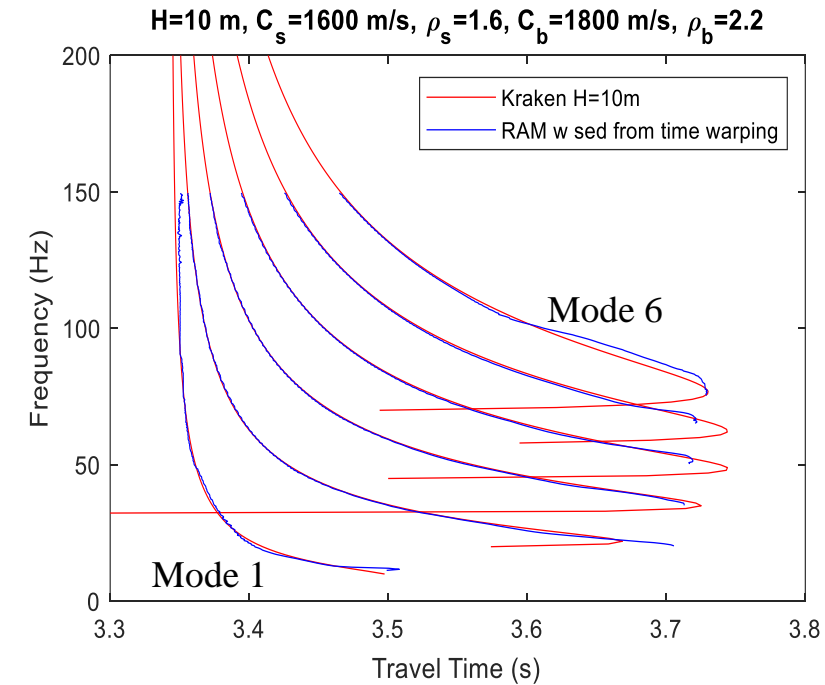
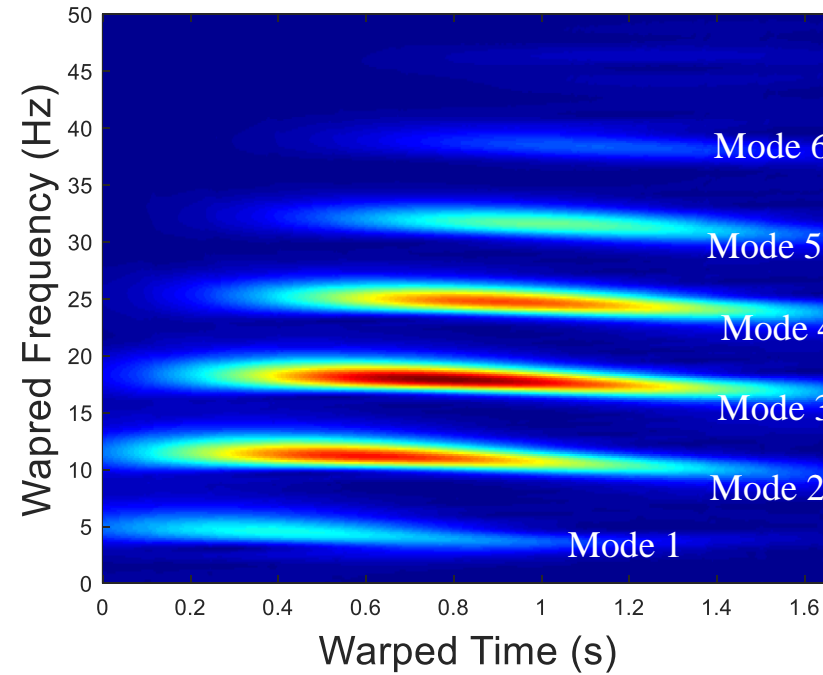
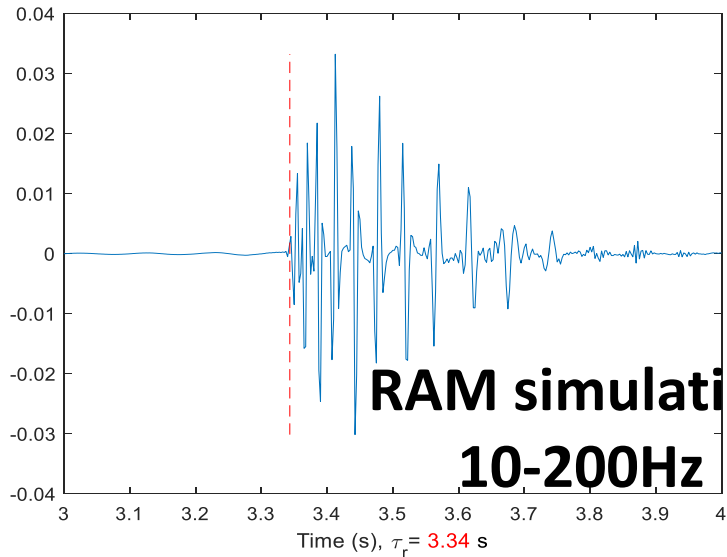
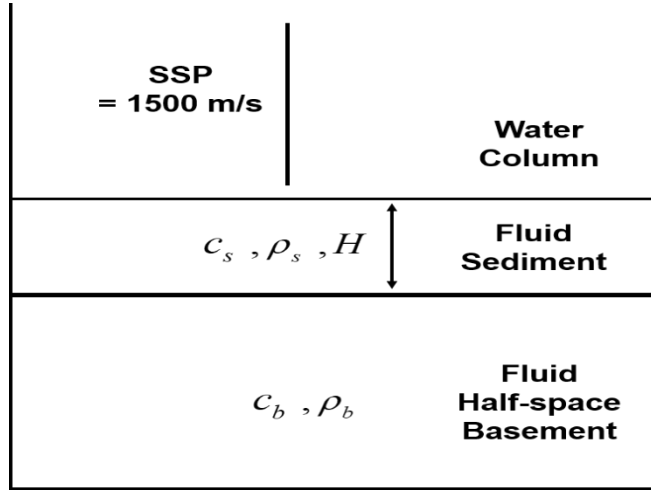
Unwarped/Restored Mode 2



# Time Warping + RAM (half-space Pekeris waveguide)

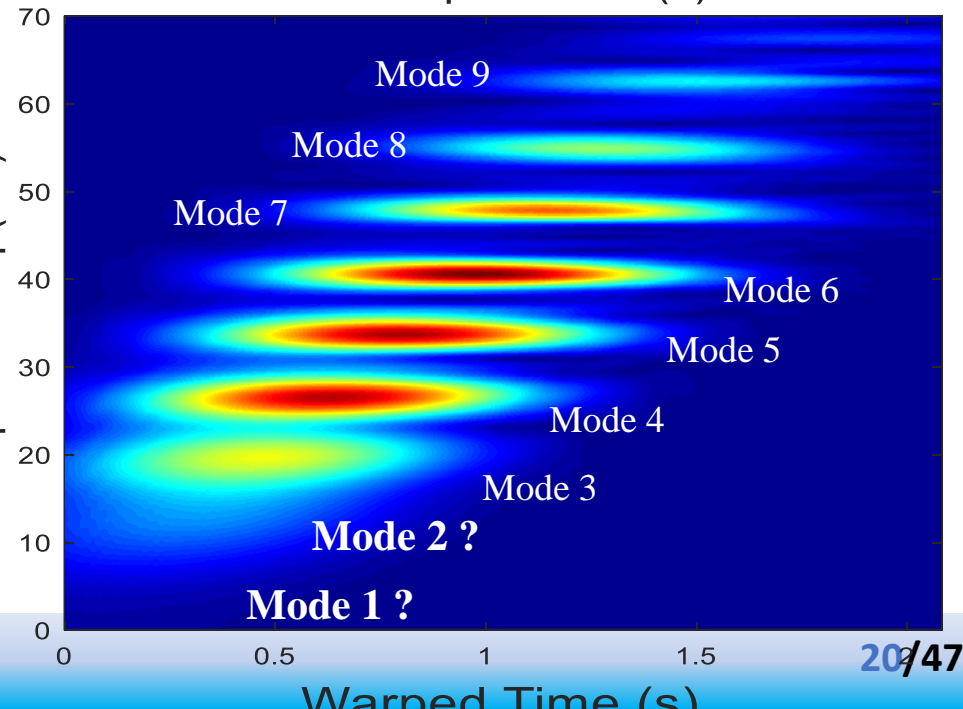
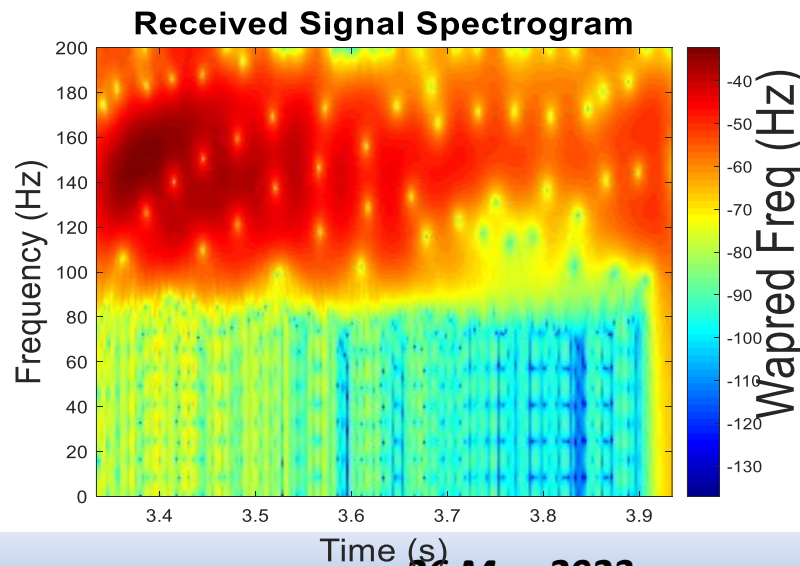
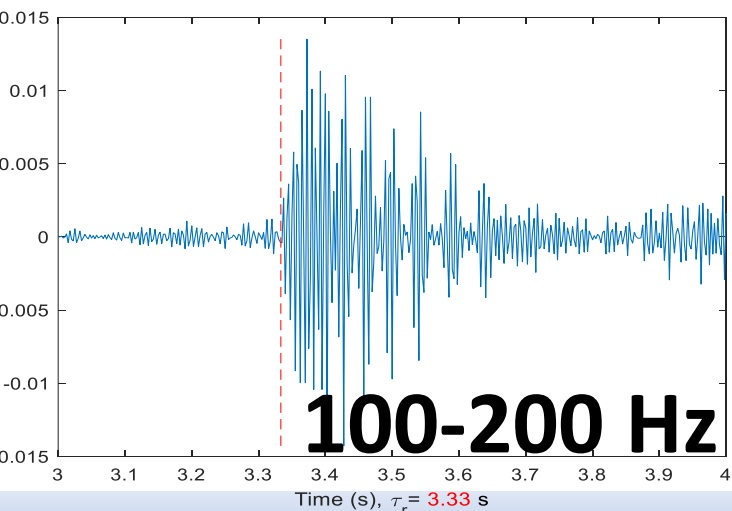
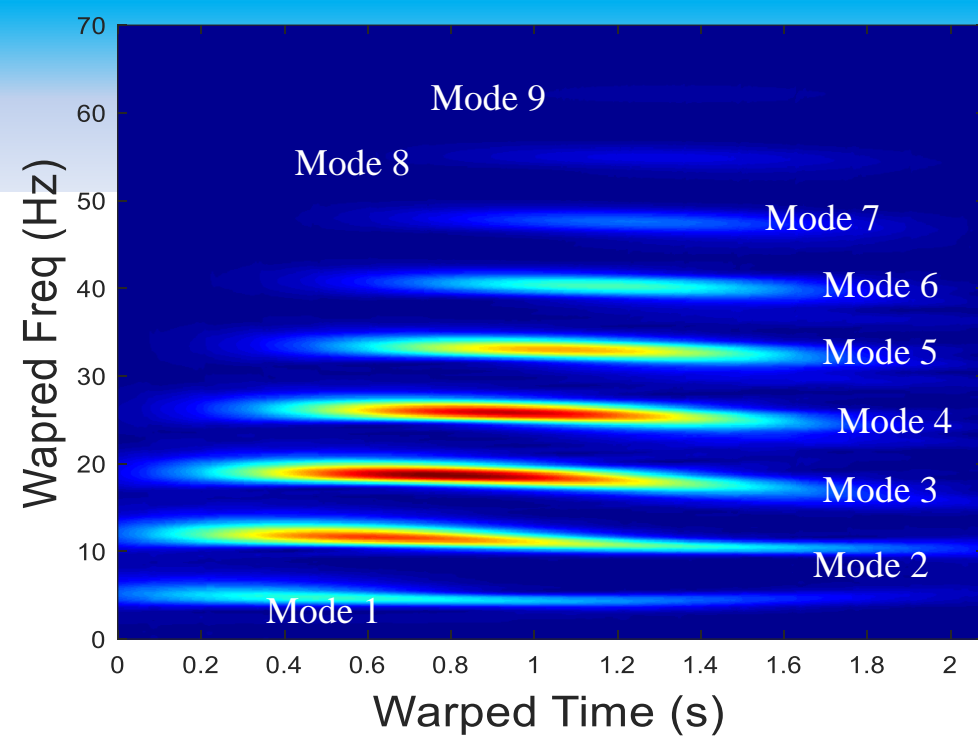
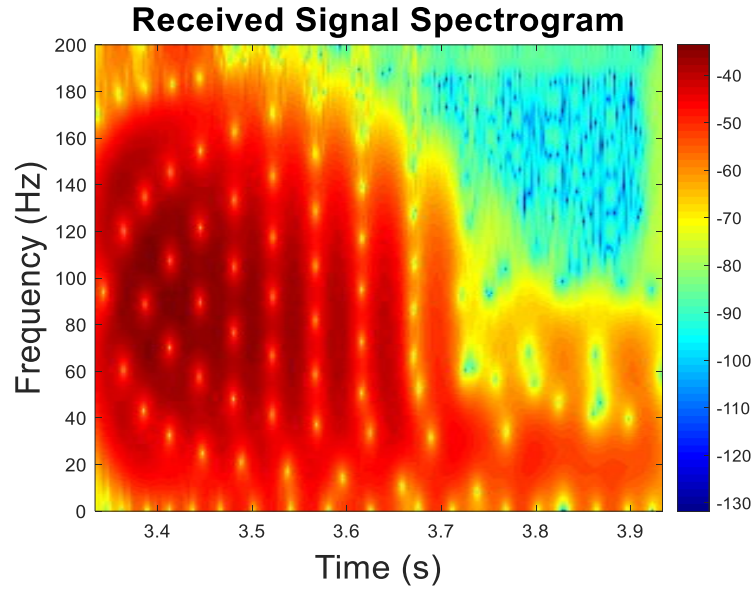
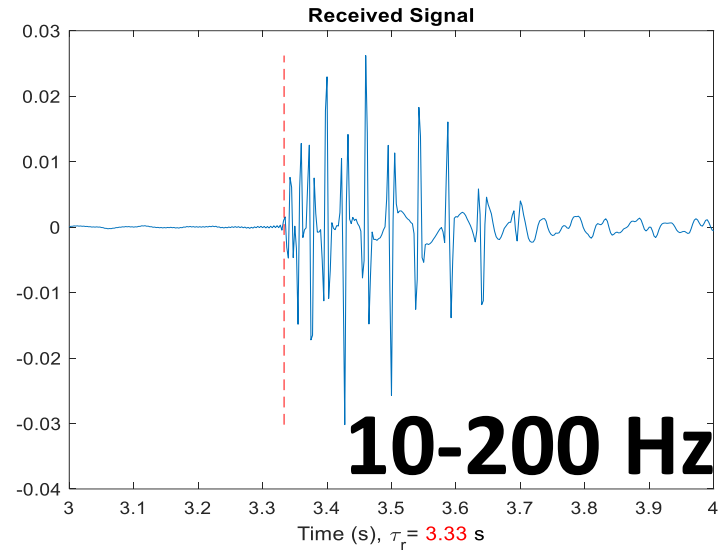


## Geoacoustic Model



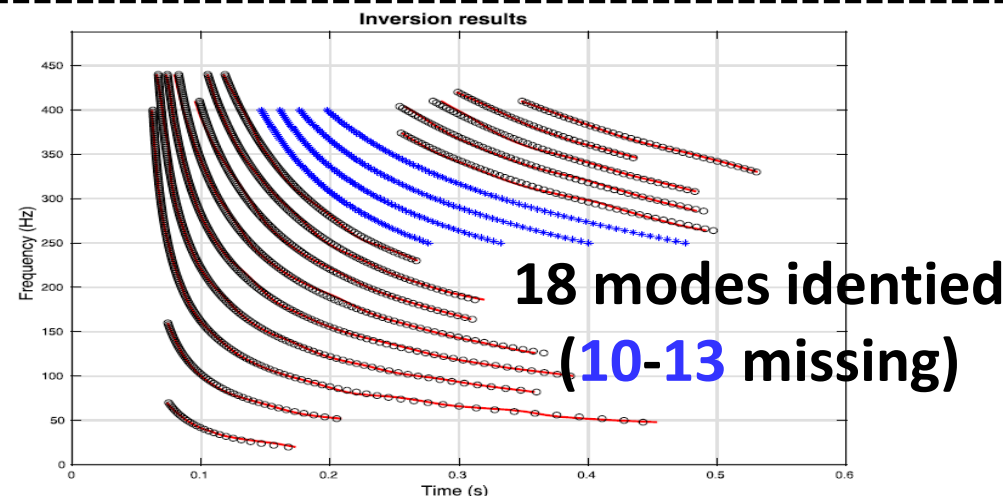
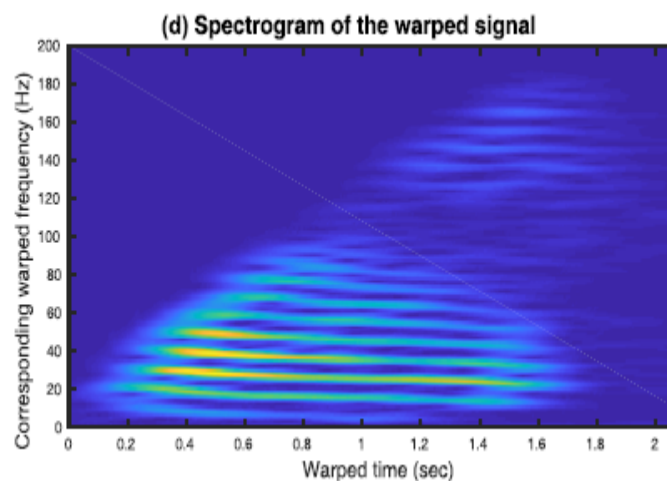
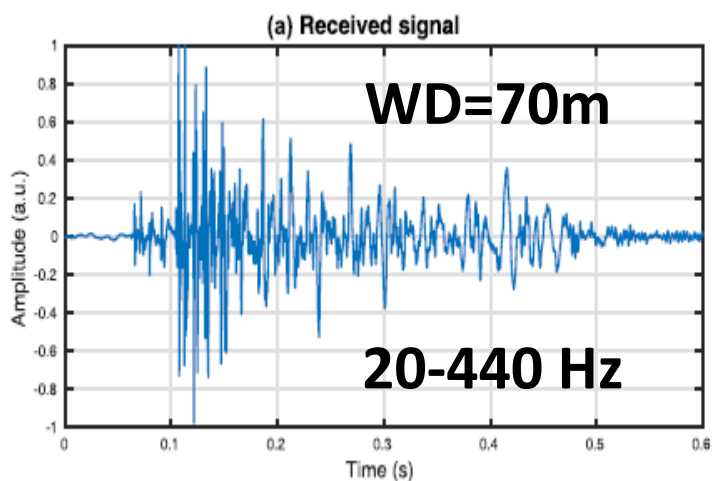
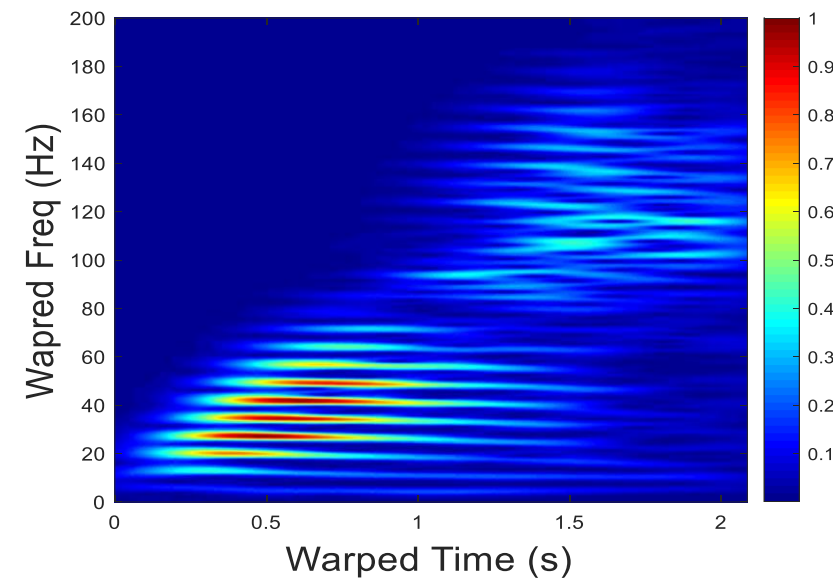
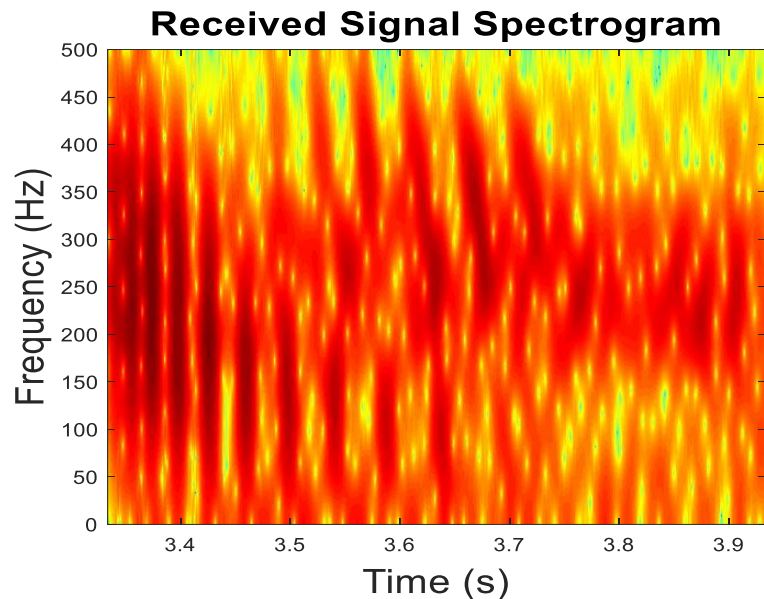
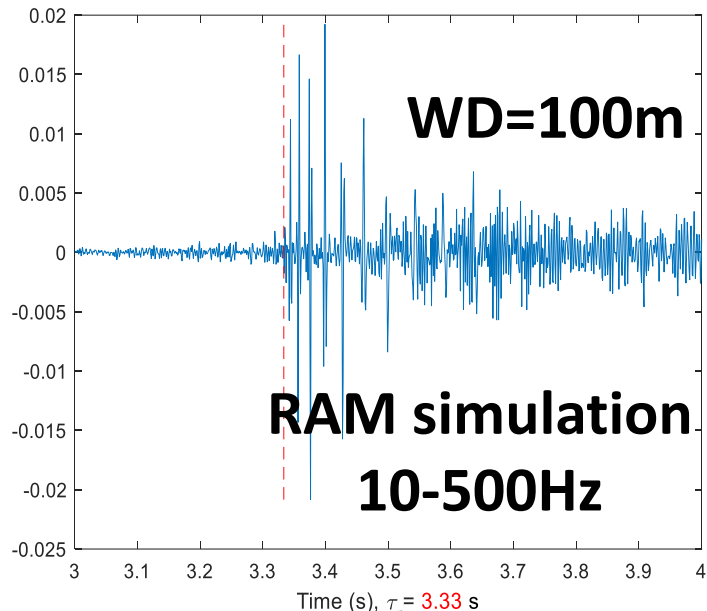
**Kraken**

# Time Warping Limitation



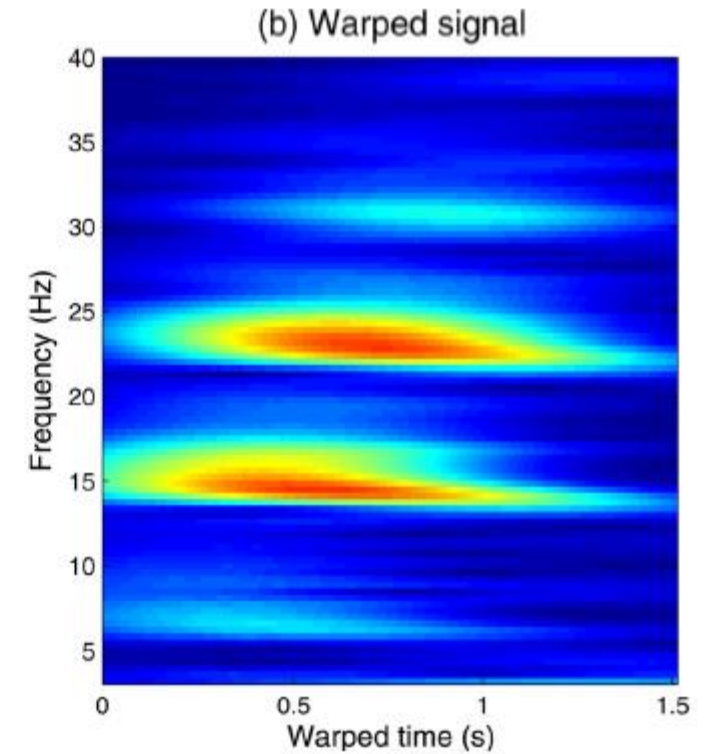
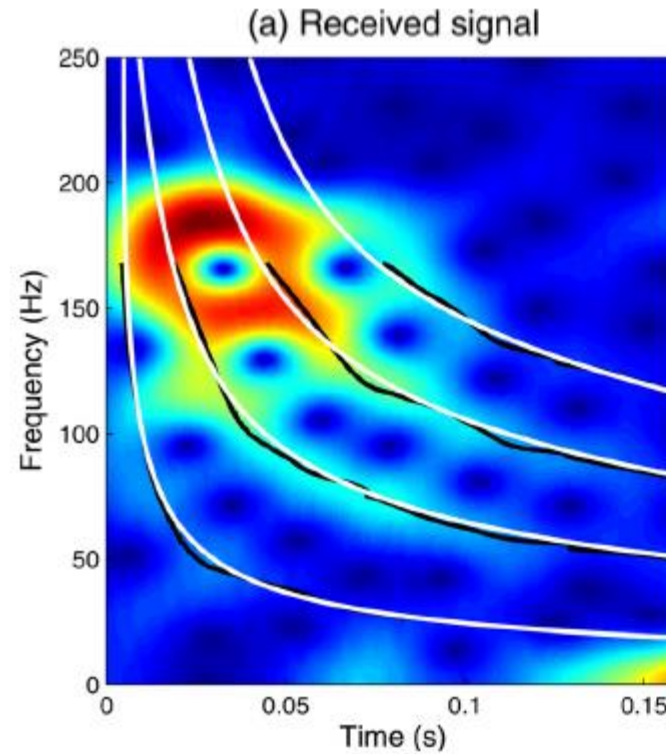
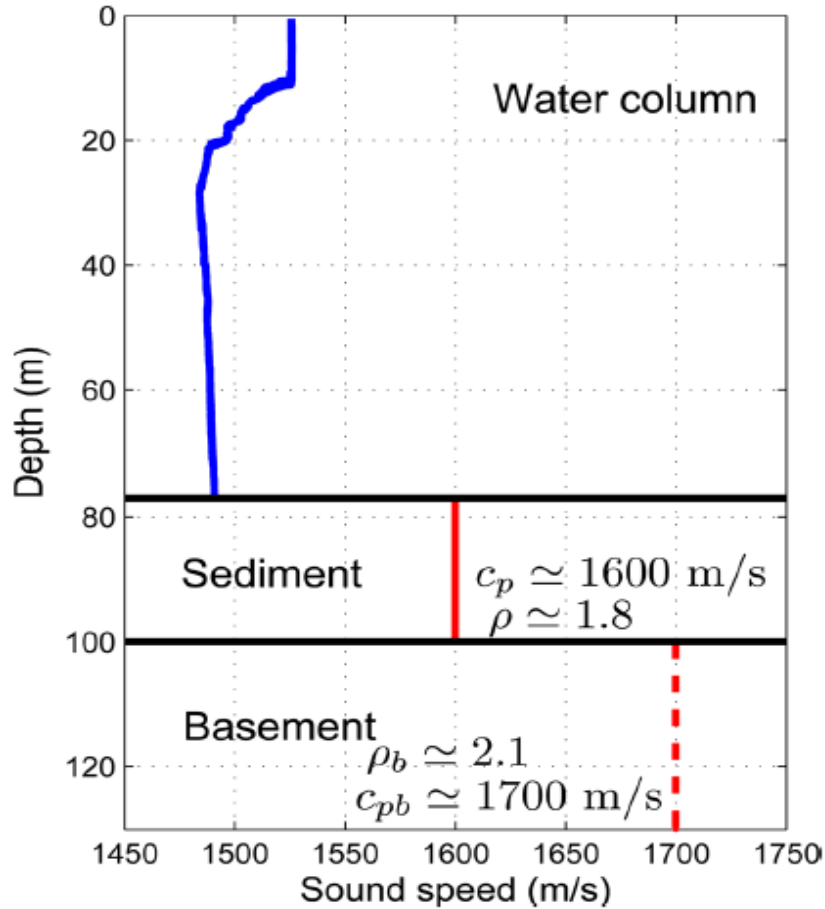


# Time Warping Application to Higher Frequency



J. Bonnel et al "Geoacoustic inversion on the New England Mud Patch using warping and dispersion curves of high-order modes" *J. Acoust. Soc. Am* 143(5), EL405-EL411(2018)

# Time Warping Applications in Underwater Acoustics (Active Scheme)



$$[\hat{A}, \hat{dt}] = \min_{A, dt} \left\{ \sum_{m,n=1}^{M,N} [\hat{t}_m(f_n) - dt - t_m(f_n, A)]^2 \right\}$$

Bonnel, Julien, and N. Ross Chapman. "Geoacoustic inversion in a dispersive waveguide using warping operators." *The Journal of the Acoustical Society of America* 130.2 (2011): EL101-EL107.

# Time Warping Application- Seabed Attenuation

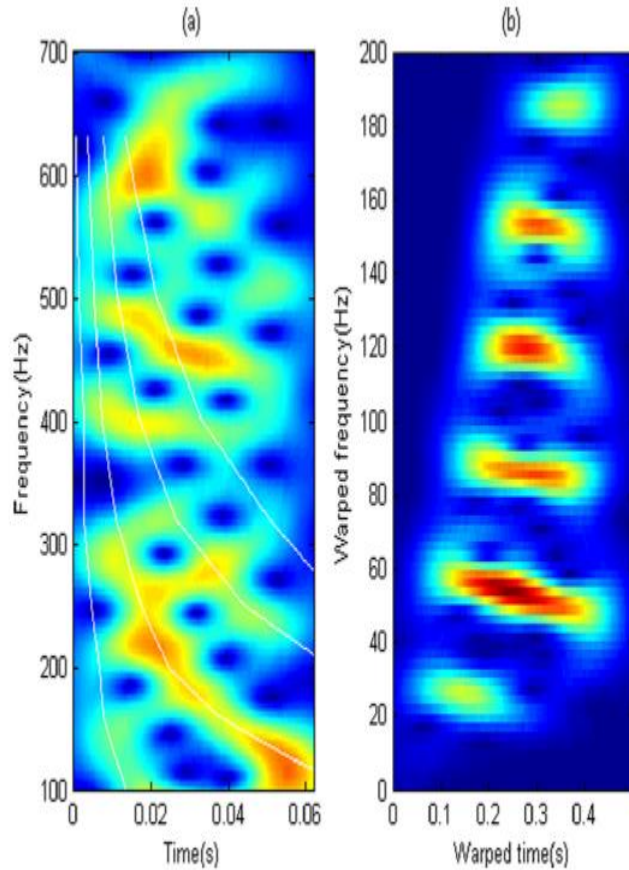


Fig. 1. (Color online) (a) Spectrogram of the original signal and the extracted dispersion curves for the first four modes, indicated by the white lines. (b) Spectrogram of the warped signal (100–700 Hz)

Zeng, J., Chapman, N.R. and Bonnel, J., 2013. **Inversion of seabed attenuation using time-warping of close range data.** *The Journal of the Acoustical Society of America*, 134(5), pp.EL394-EL399.

$$P(\omega, z_r) \approx BS(\omega) \sum_{m=1}^M \psi_m(\omega, z_s) \psi_m(\omega, z_r) \frac{e^{jk_m(\omega)r - \beta_m(\omega)r}}{\sqrt{k_m(\omega)r}}$$

$$A_m(\omega) = [\psi_m(z_s)\psi_m(z_r)/k_m]e^{-\beta_m r} / \sqrt{\sum_{m=0}^M \left| \frac{\psi_{m0}(z_s)\psi_{m0}(z_r)}{k_{m0}} e^{-\beta_{m0}r} \right|^2}$$

$$A0_m = \sqrt{\sum_{\omega_w = \omega_w^m - \Delta\omega_w}^{\omega_w^m + \Delta\omega_w} |F_w(\omega_w)|^2}$$

$$A_m = A0_m / \sqrt{\sum_{m=1}^M A0_m^2}$$

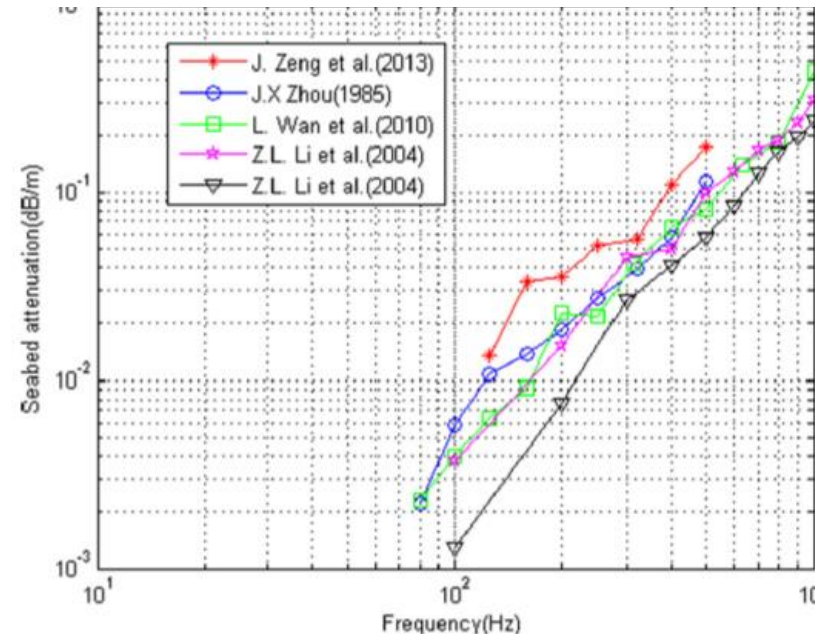
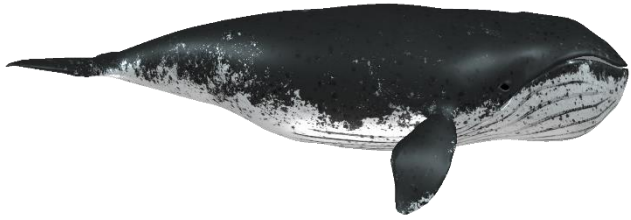


Table 2. Estimates of the seabed attenuation.

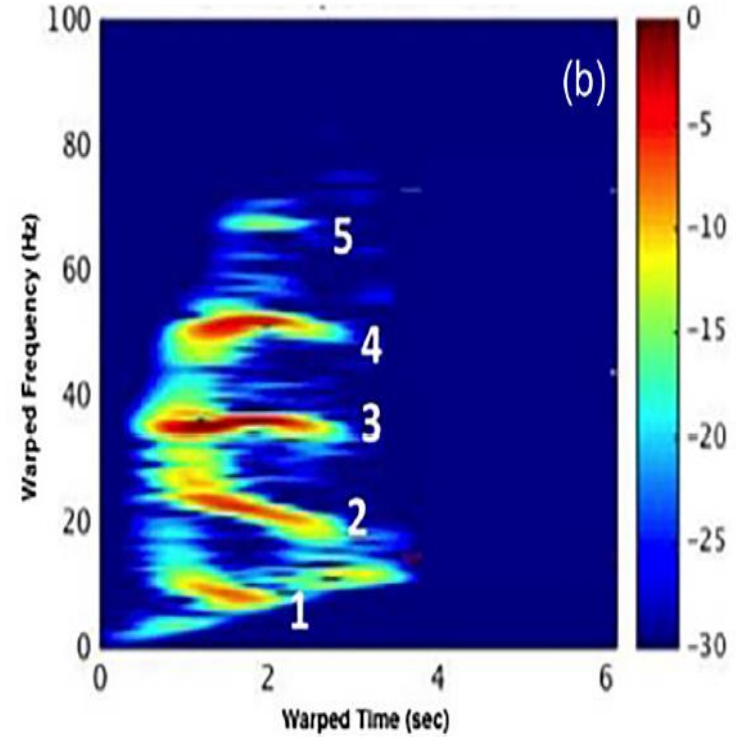
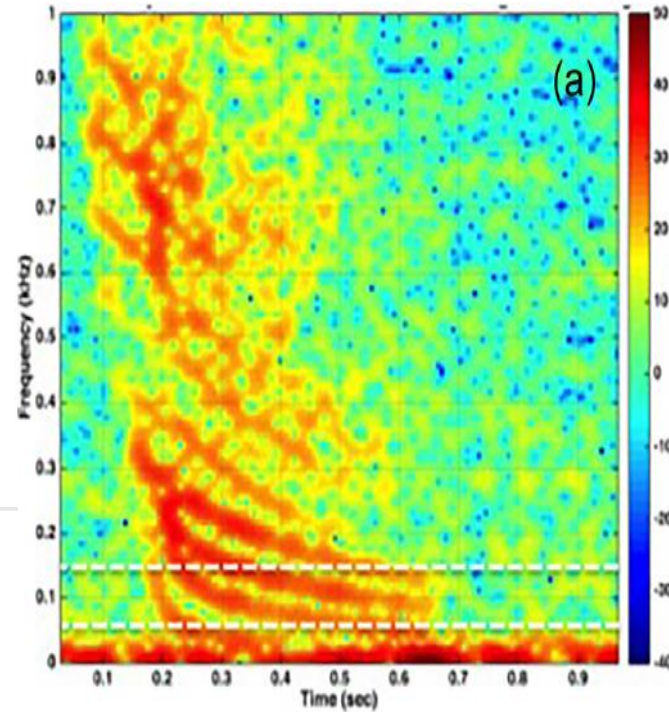
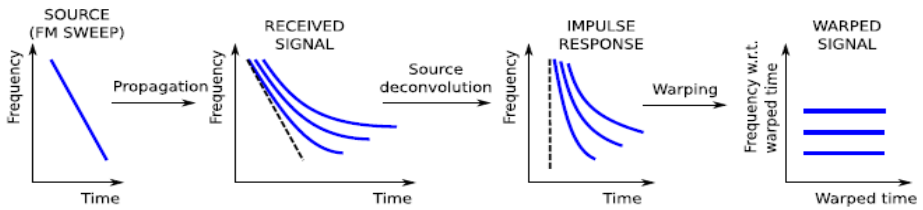
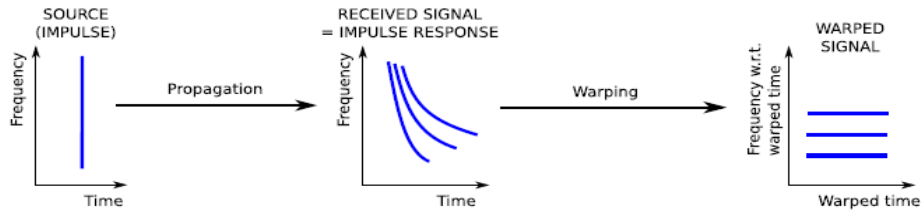
Frequency (Hz)	125	160	200	250	320	400	500
Attenuation (dB/m)	0.0134	0.0334	0.0352	0.0522	0.0567	0.109	0.174

$$\hat{\alpha}_b = \min_{\alpha_b} \left\{ \sum_{m=1}^M [\hat{A}_m(\omega) - A_m(\omega, \alpha_b)]^2 \right\}$$





北太平洋露脊鯨  
(North Pacific right whale)



Thode, Aaron, et al. "Using nonlinear time warping to estimate North Pacific right whale calling depths in the Bering Sea." The Journal of the Acoustical Society of America 141.5 (2017): 3059-3069.

Bonnel, J., Thode, A., Blackwell, S., Kim, K., and Macrander, A. (2014). "Range estimation of bowhead whale (Balaena mysticetus) calls in the arctic using a single hydrophone," J. Acoust. Soc. Am. 136(1), 145–155

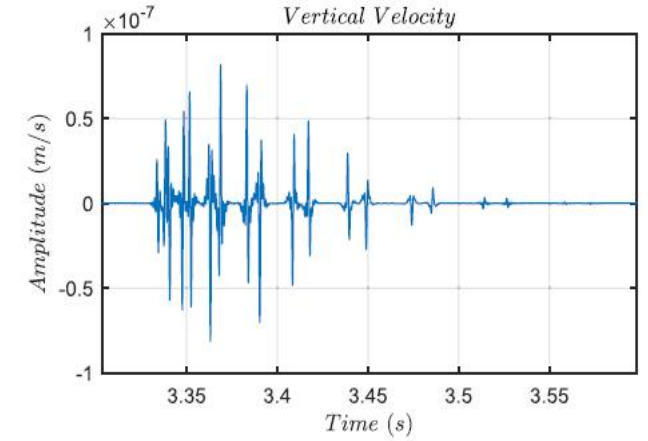
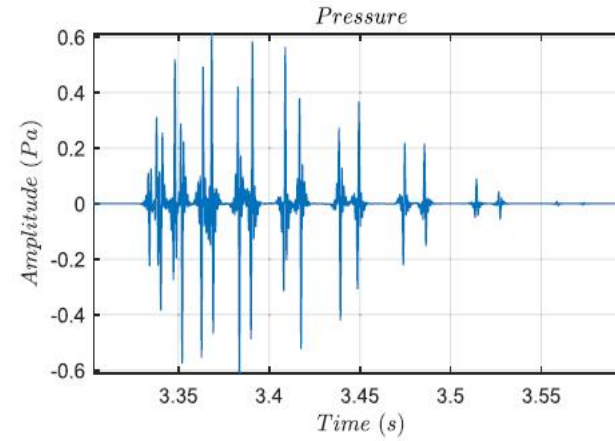
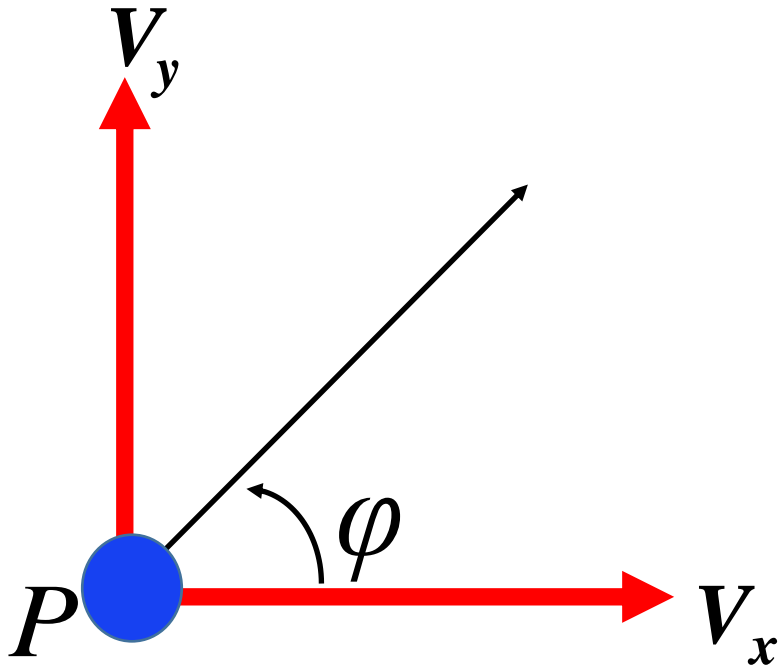
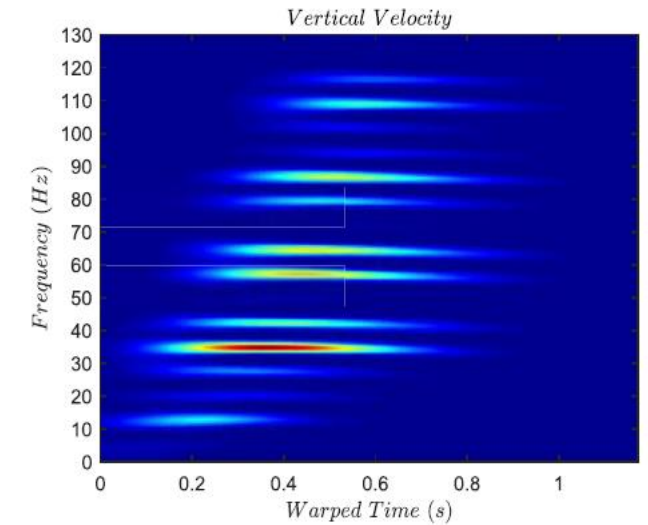
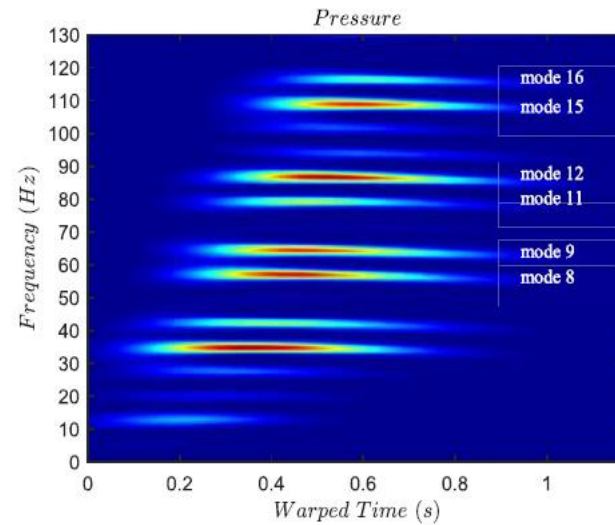


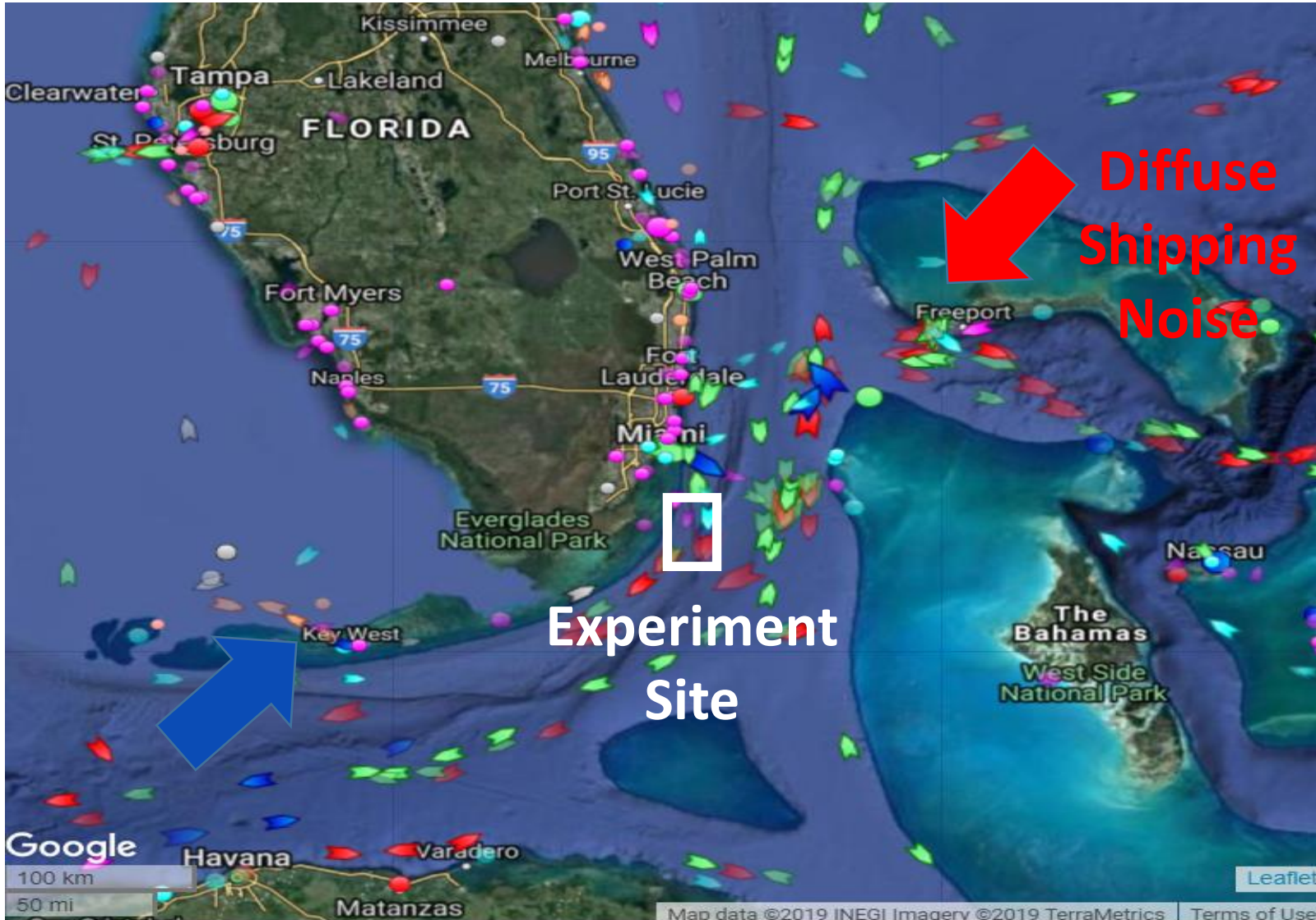
Fig. 11. Theoretical pressure and vertical velocity signals at the vector sensor.



Guarino, A. L., Smith, K. B., & Godin, O. A. (2022). Bottom attenuation coefficient inversion based on the modal phase difference between **pressure and vertical velocity from a single vector sensor**. *Journal of Theoretical and Computational Acoustics*, 30(02), 2150008.

# **Time Warping on Noise Cross-Correlation Function (NCCF) Retrieved from Florida Straits 2012 Experiment**

# FL Straits Shipping Lanes



Diffuse  
Shipping  
Noise

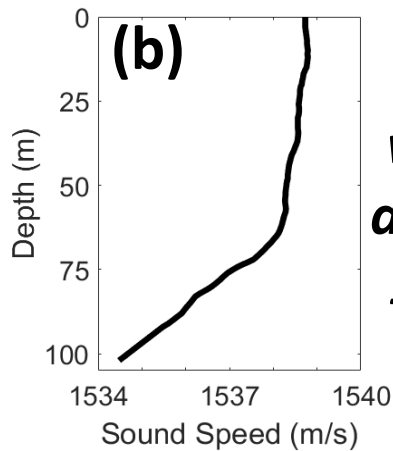
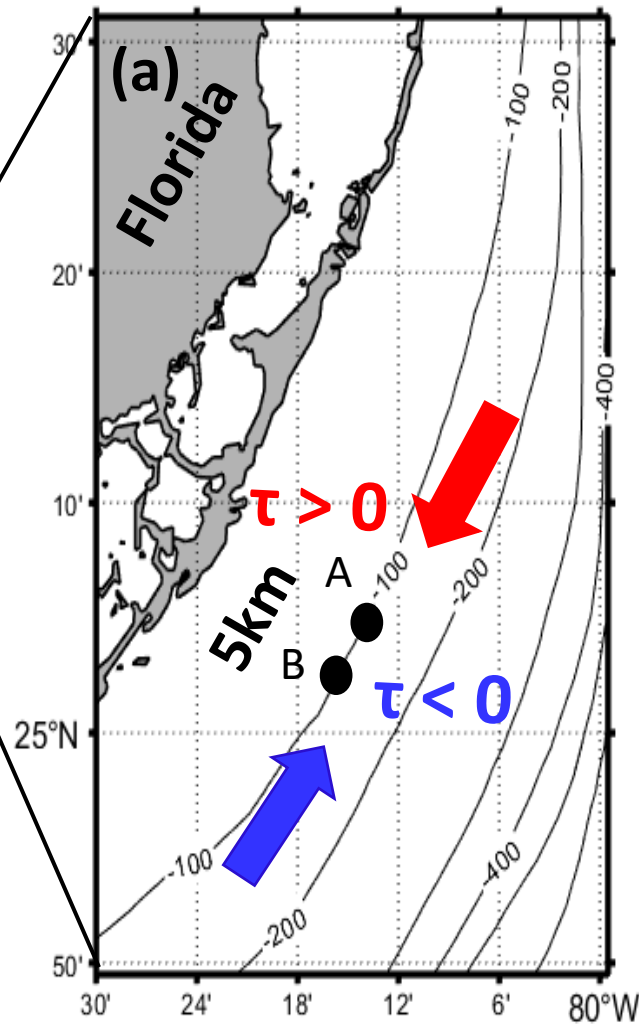
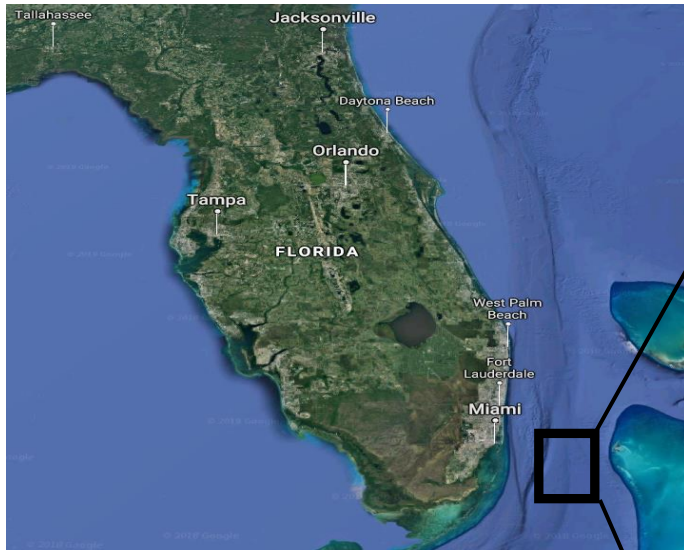
Diffuse  
Shipping  
Noise

Experiment  
Site

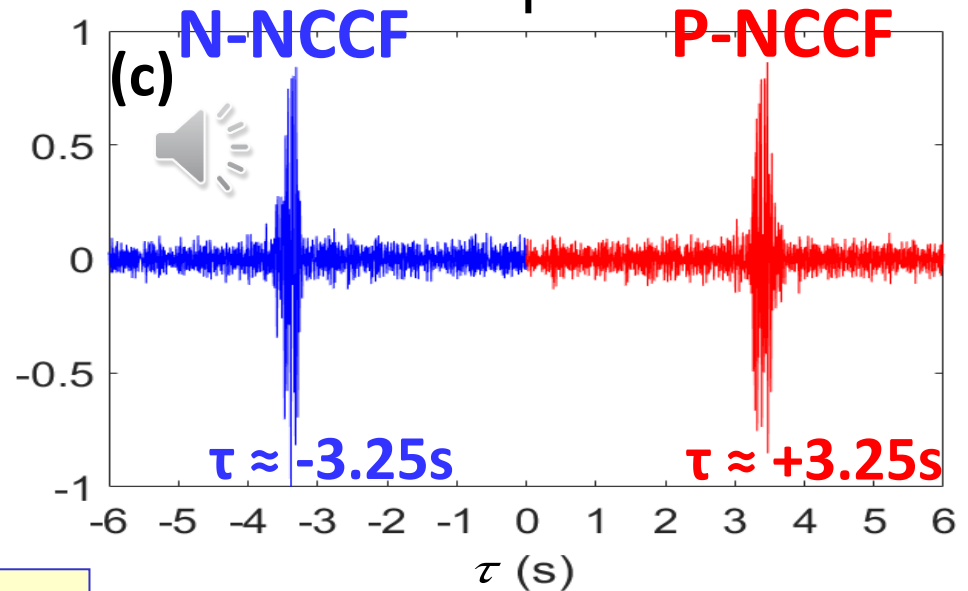
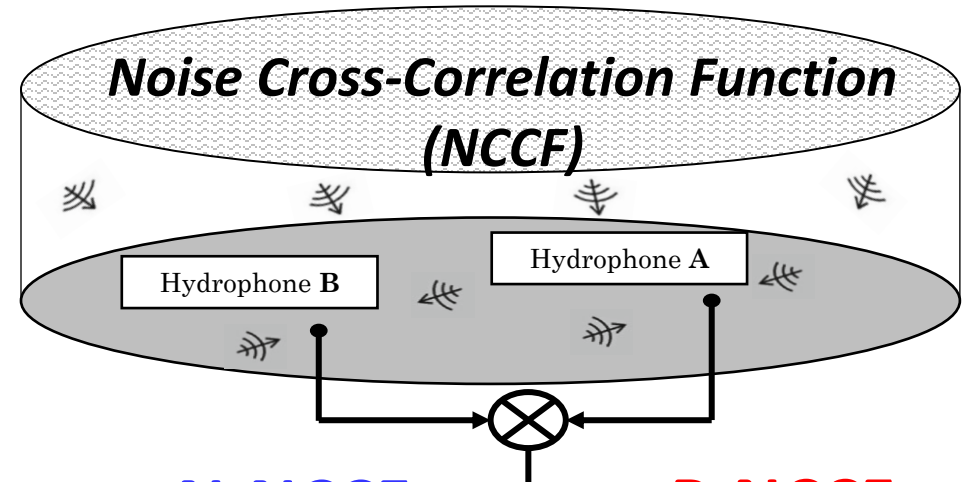
<http://www.shiptraffic.net>



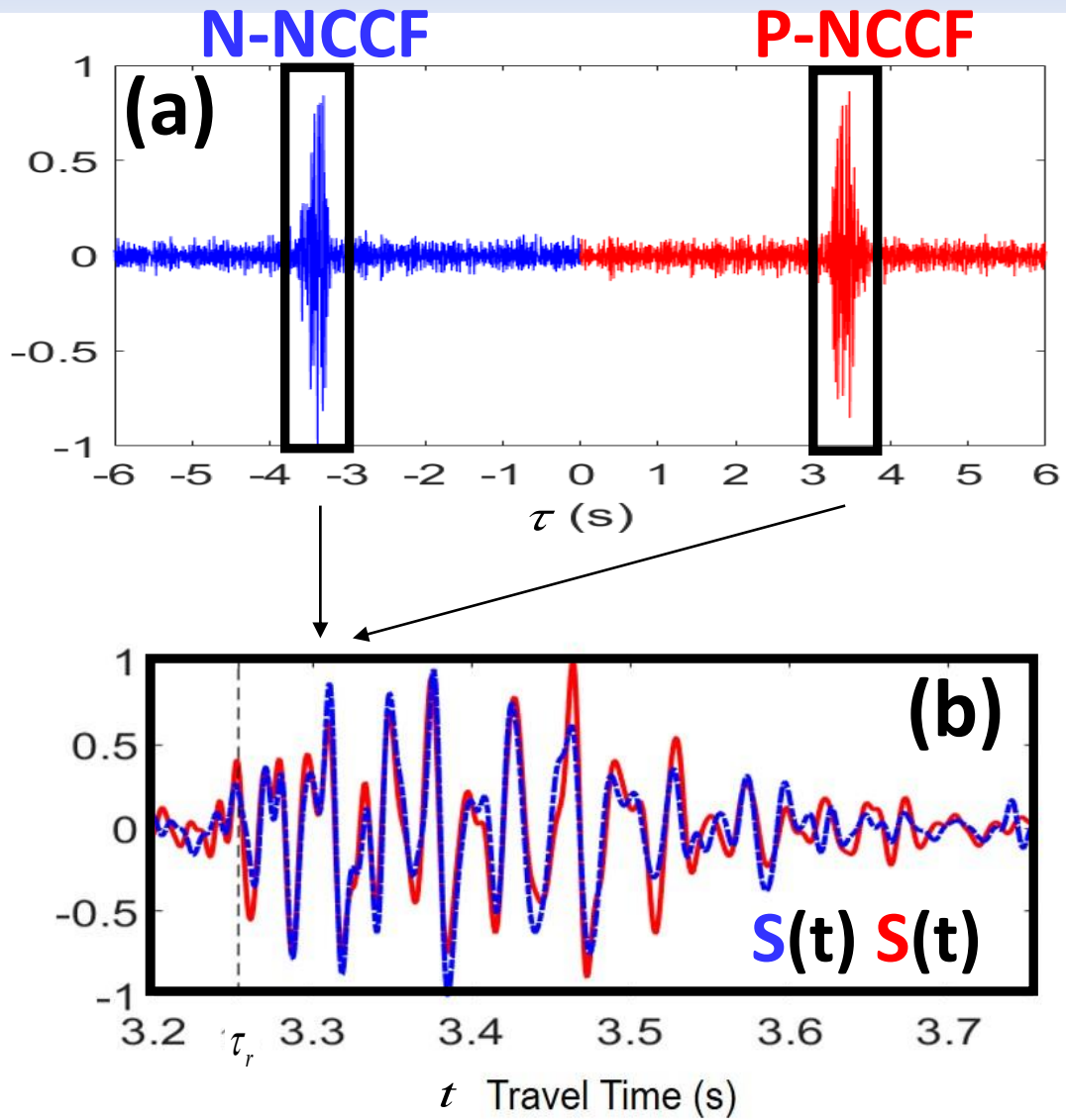
# Florida Straits 2012 NI Experiment & NCCF



SSP  
weakly  
dynamic  
 $\pm 2$  m/s



T. Tan, O.A. Godin, M.G. Brown, and N.A. Zaboltn, "Characterizing the seabed in the Straits of Florida by using acoustic noise interferometry and time warping," J. Acoust. Soc. Am. 146(4), 2321-2334 (2019)



## NCCF as Approximated Green's function

$$\frac{d}{dt} C_{AB}(\tau) \cong [G(\mathbf{r}_B, \mathbf{r}_A, -\tau) - G(\mathbf{r}_A, \mathbf{r}_B, \tau)]$$

## Warping/Unwarping Function

$$w(t) = \sqrt{t^2 - \tau_r^2} \quad w^{-1}(t) = \sqrt{t^2 + \tau_r^2}$$

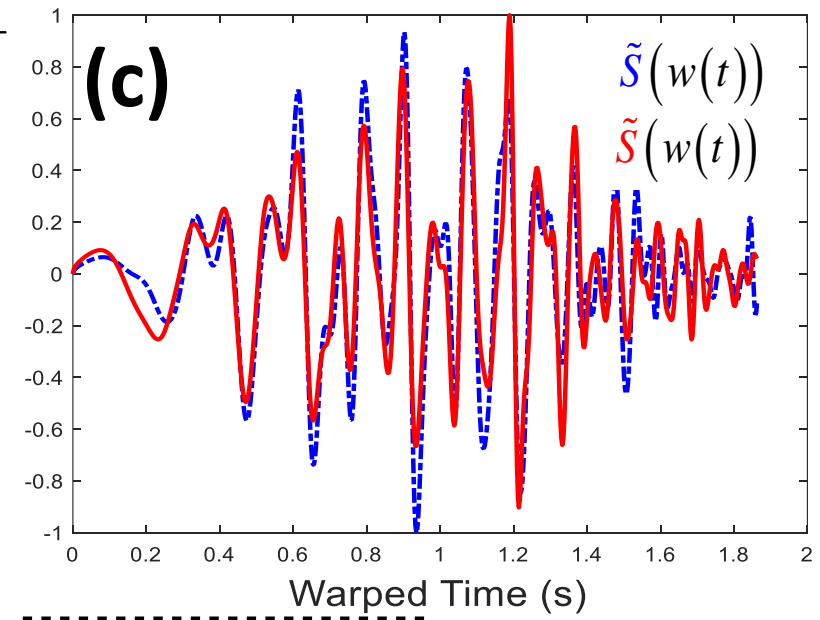
**(b) Warping (c)**

$$|w'(t)|^{-1/2} S(t) = \tilde{S}(w(t))$$

$$|w'(t)|^{-1/2} \tilde{S}(w(t)) = S(t)$$

**(b) Unwarping (c)**

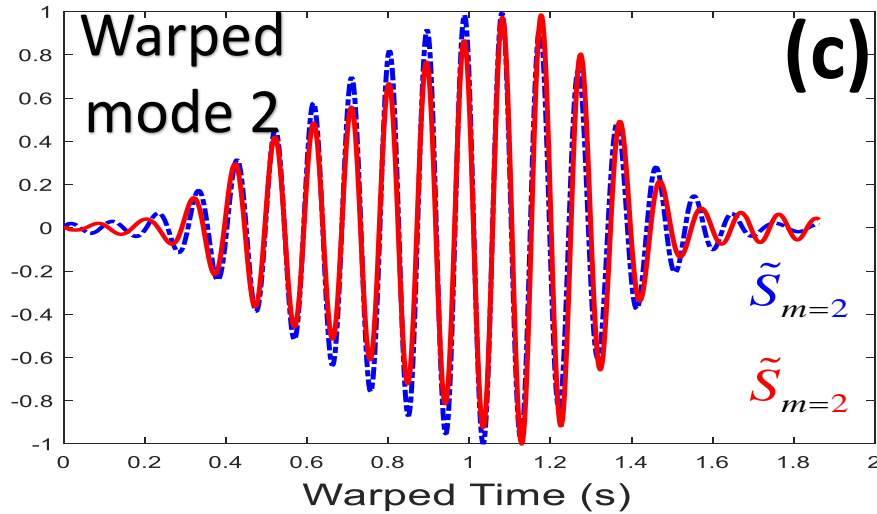
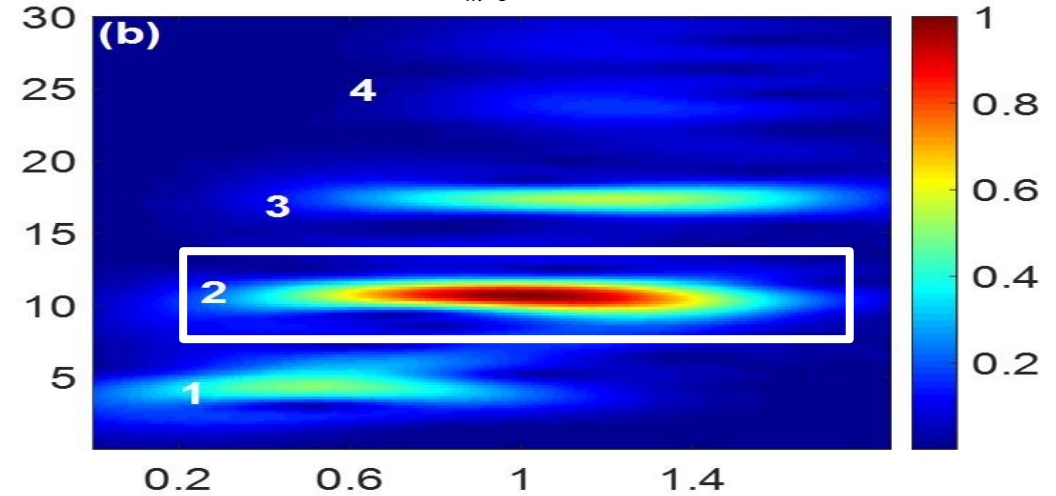
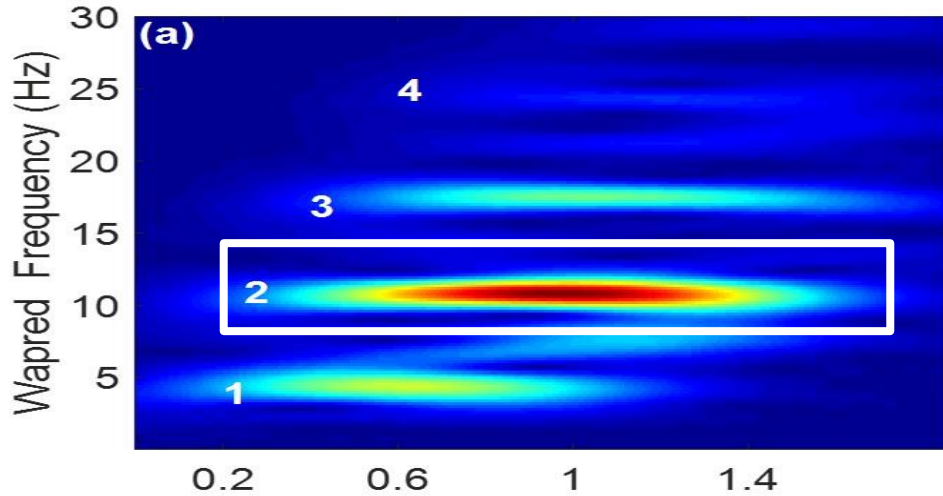
$$w^{-1}[\tilde{S}(w(t))] = S(t)$$



# N-NCCF & P-NCCF mode restoration

$$\tilde{S}(w(t)) = \sum_{m=1}^4 \tilde{S}_m(w(t))$$

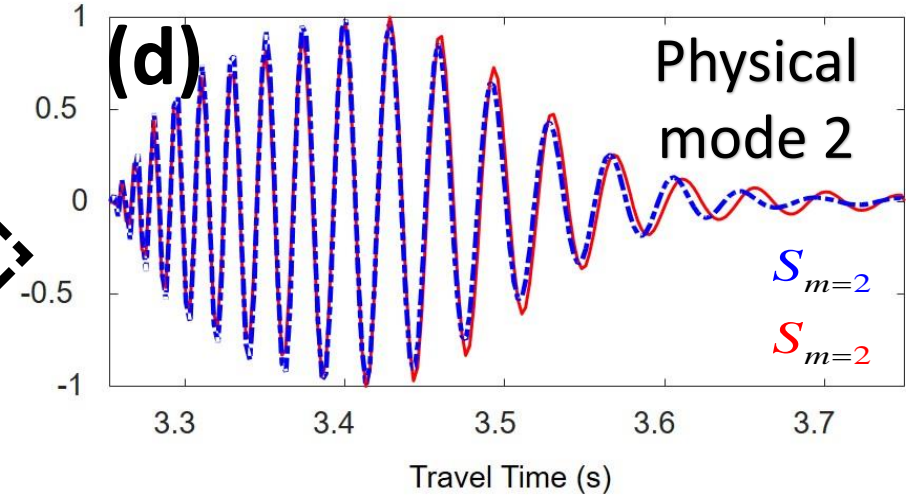
$$\tilde{S}(w(t)) = \sum_{m=1}^4 \tilde{S}_m(w(t))$$



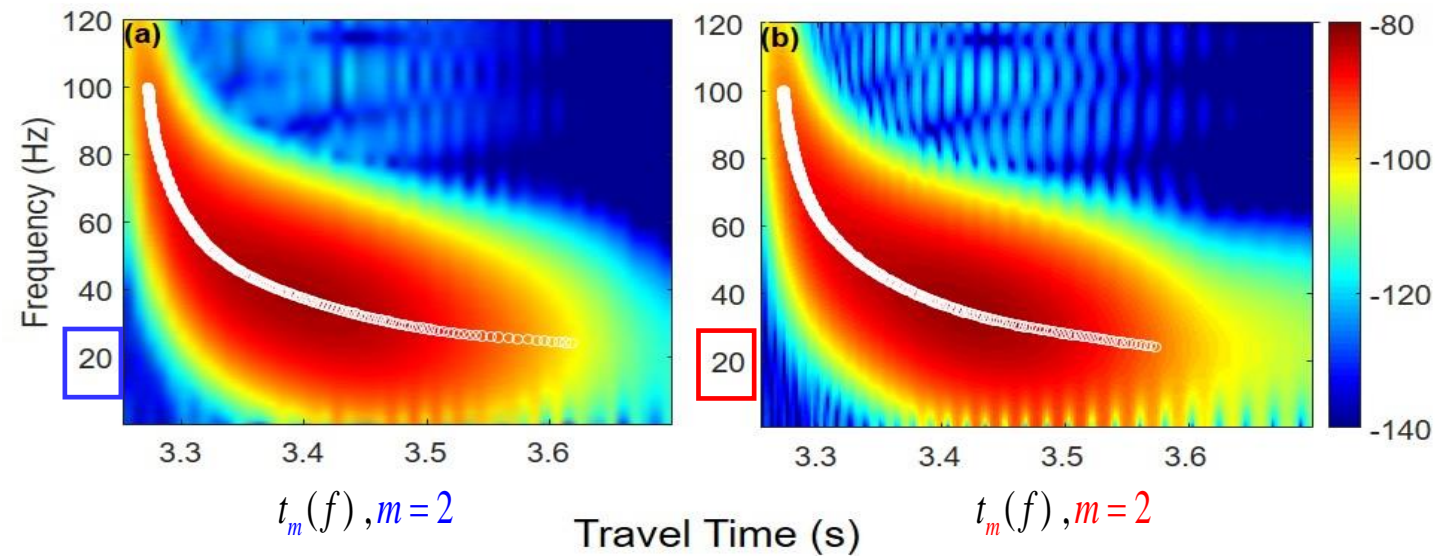
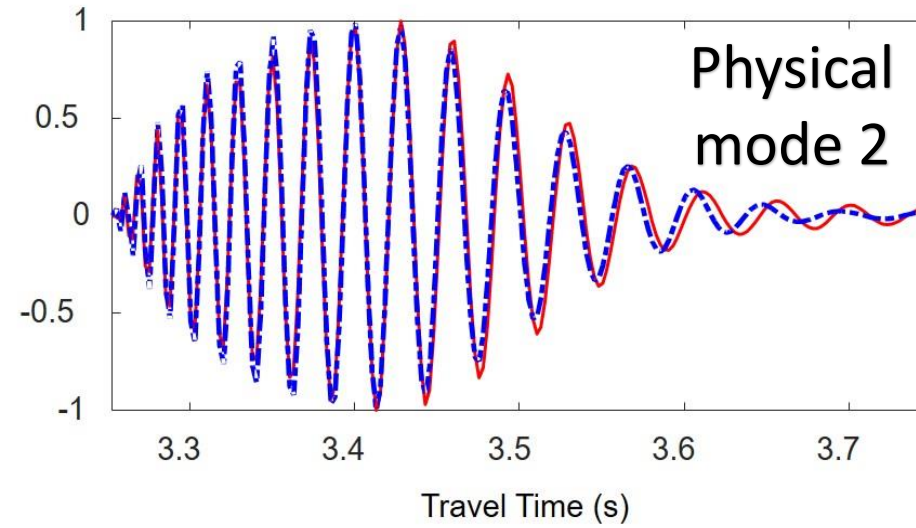
$$w^{-1}(t) = \sqrt{t^2 + \tau_r^2}$$



$$w^{-1}[\tilde{S}_m(w(t))] = S_m(t), m=2$$

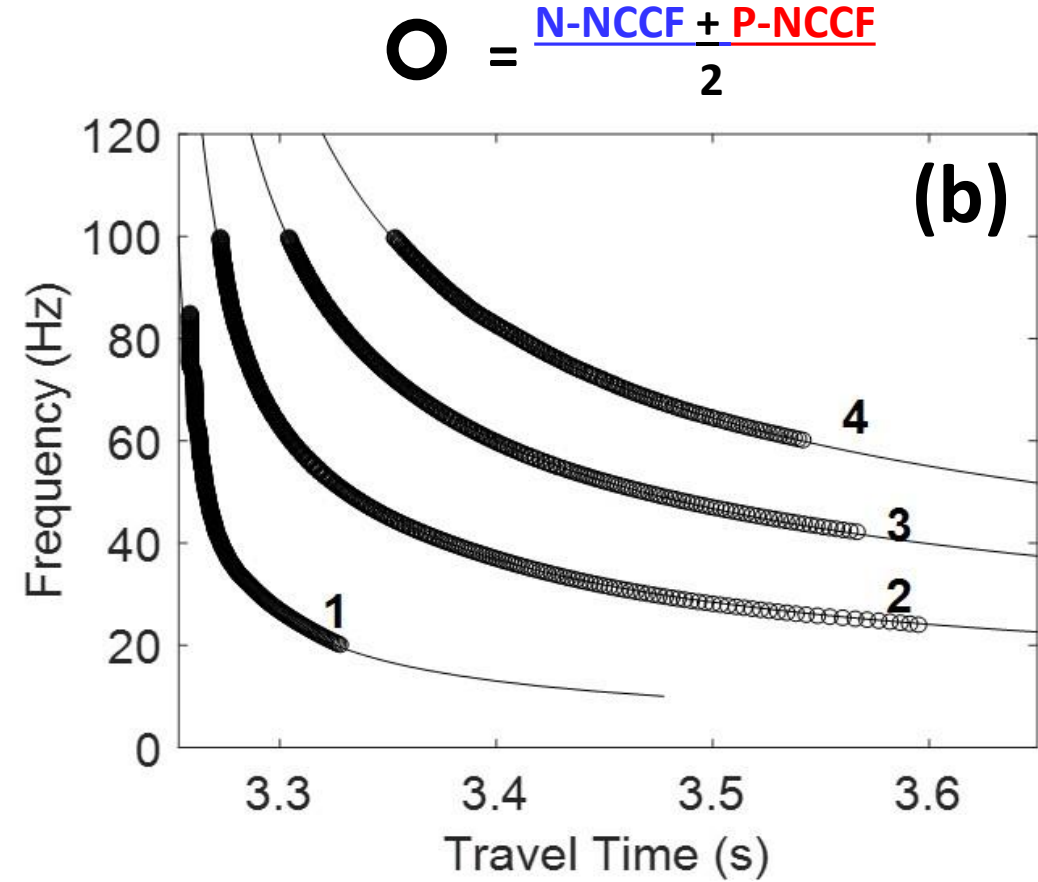
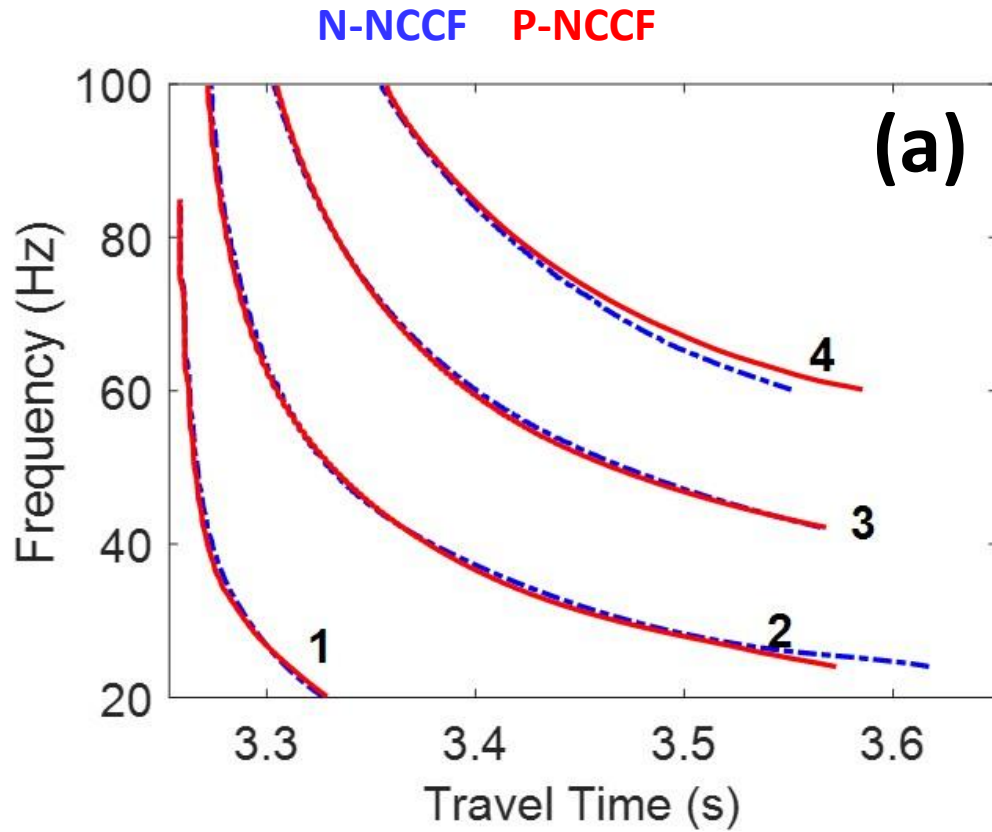


# Retrieving N-NCCF & P-NCCF mode dispersion curve





# Inverse problem



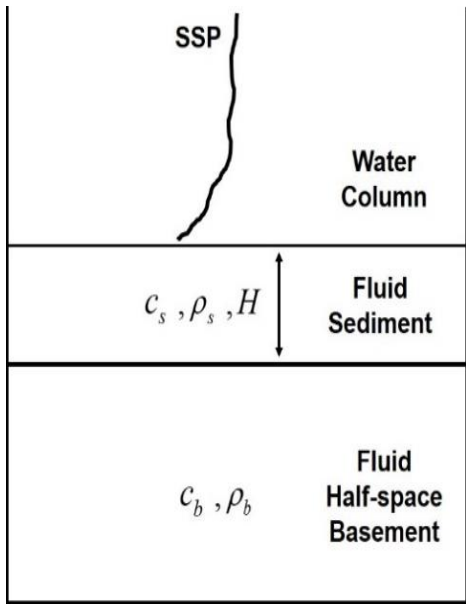
$$\text{Cost function: } K(\mathbf{U}) = \frac{1}{M} \sum_{m,n=1}^{M=4,N} \left[ \frac{t_m(f_n) - \hat{t}_m(f_n, \mathbf{U})}{N} \right]^2$$



# 1-D Sensitivity Animation



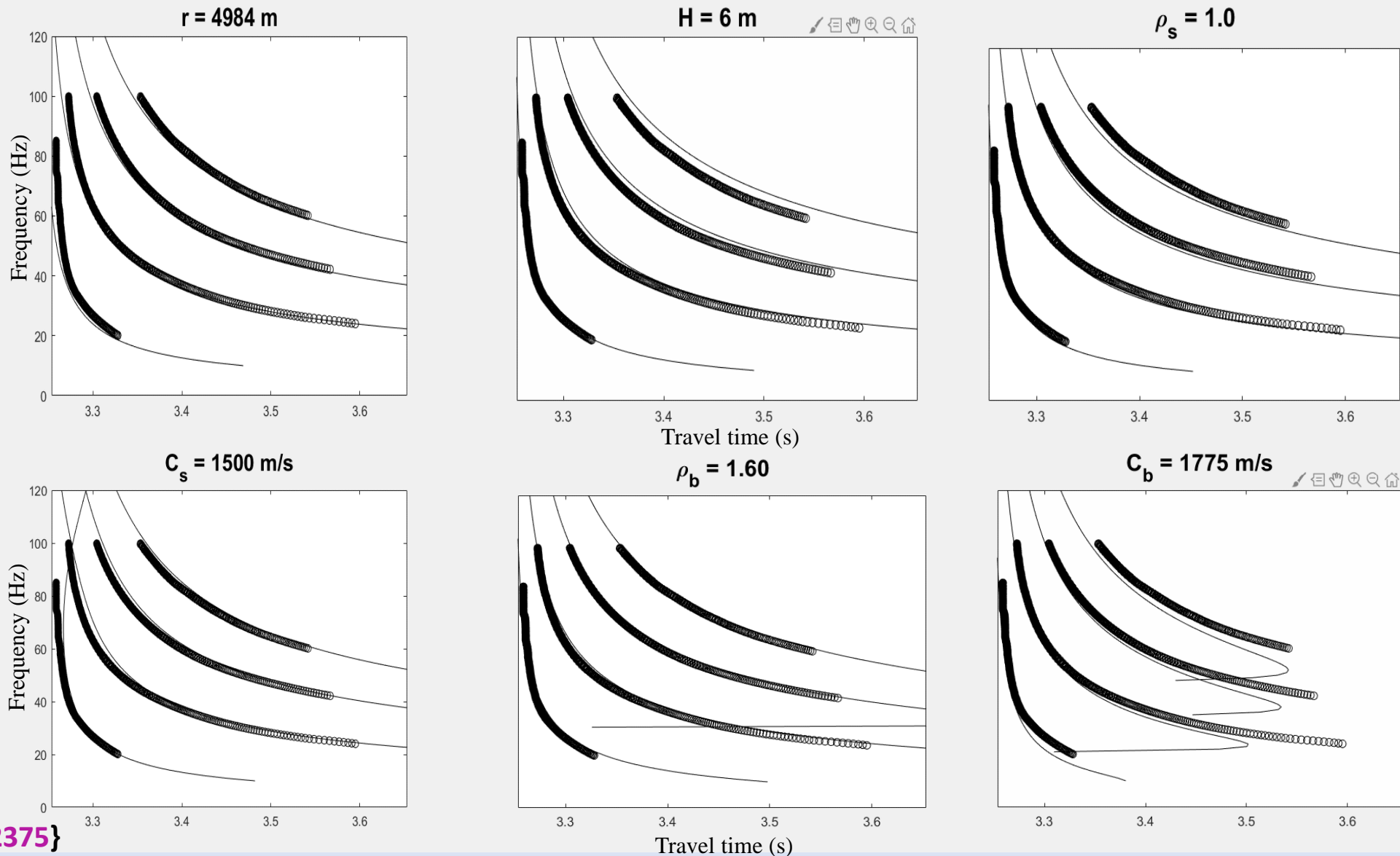
## Geoacoustic model



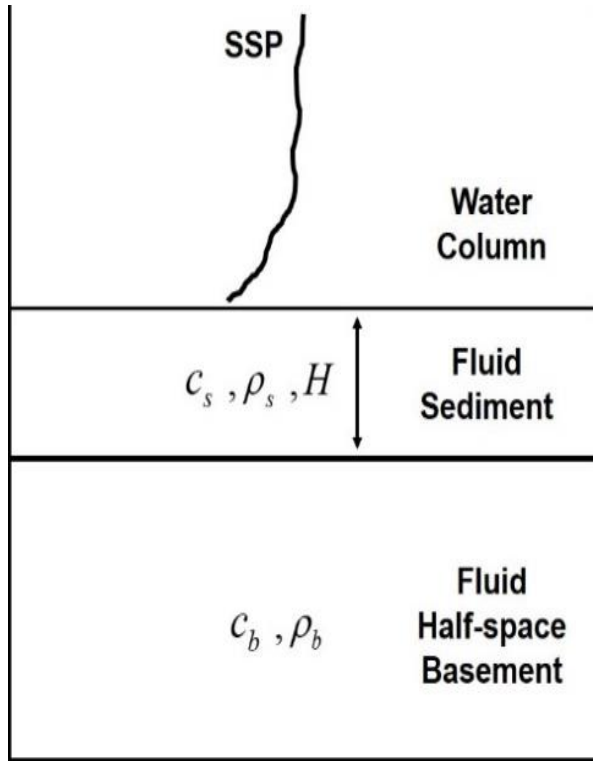
- $r$  : distance btw receivers
- $H$  : sediment thickness
- $\rho_s$  : sediment density
- $C_s$  : sediment sound speed
- $\rho_b$  : basement density
- $C_b$  : basement sound speed

$$U = \{r, H, \rho_s, C_s, \rho_b, C_b\}$$

$$U = \{4994, 14, 1.4, 1550, 2.35, 2375\}$$



## (a) Geoacoustic model

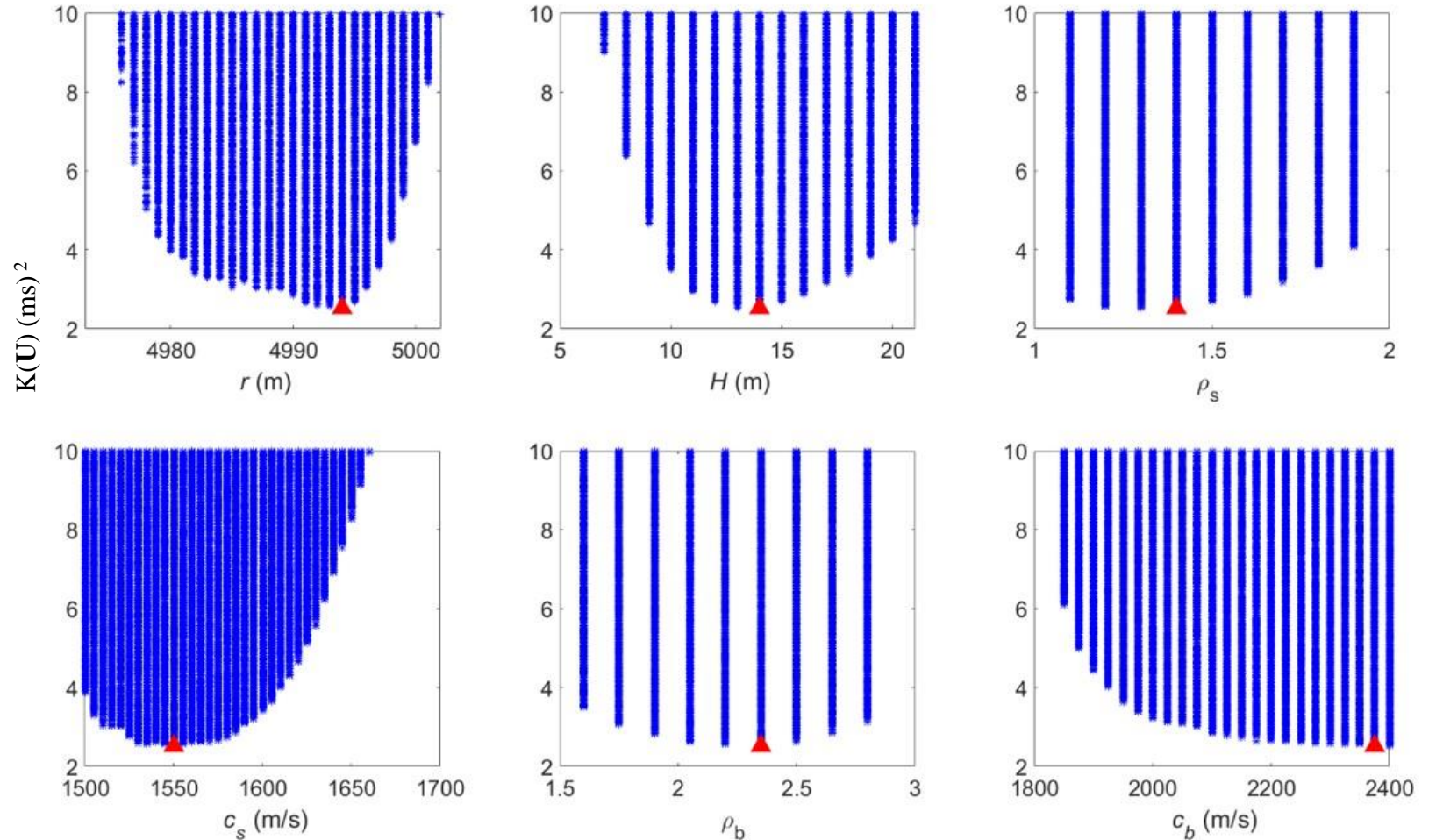


$$K(\mathbf{U}) = \frac{1}{M} \sum_{m,n=1}^{M=4,N} \left[ \frac{t_m(f_n) - \hat{t}_m(f_n, \mathbf{U})}{N} \right]^2$$

$$\mathbf{U} = \{r, H, c_s, c_b, \rho_s, \rho_b\}$$

## (b) Sensitivity Analysis

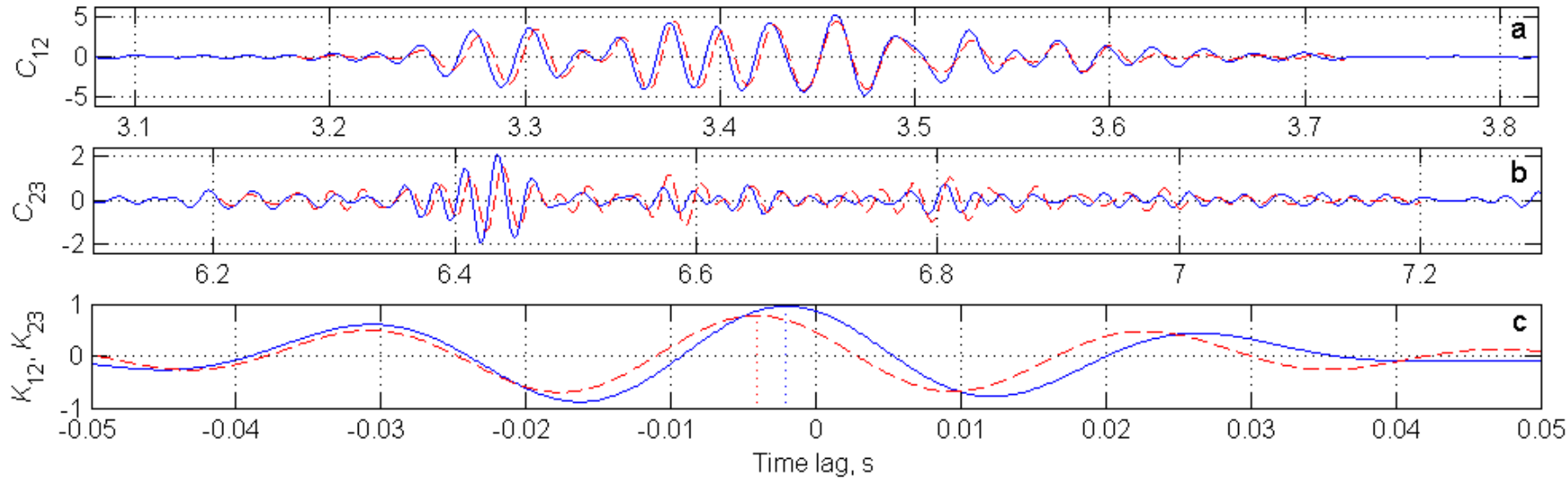
$$\blacktriangle = K(\hat{\mathbf{U}}) \approx 2.52 \text{ (ms)}^2$$



T. Tan, O.A. Godin, M.G. Brown, and N.A. Zaboltn, "Characterizing the seabed in the Straits of Florida by using acoustic noise interferometry and time warping," *J. Acoust. Soc. Am.* 146(6), 2321-2334 (2019).

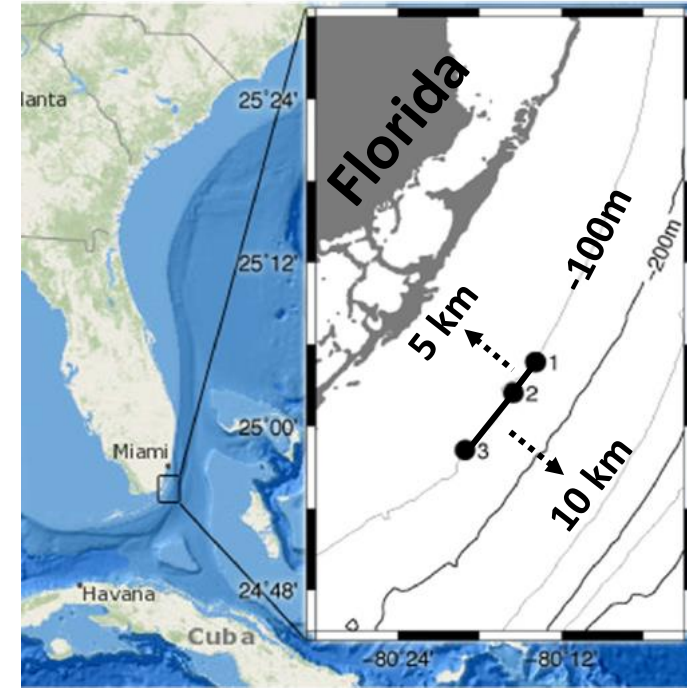
# FL Straits Experiment Future Work

# Nonreciprocity of **N-NCCF** & **P-NCCF** to estimate depth-averaged flow velocity



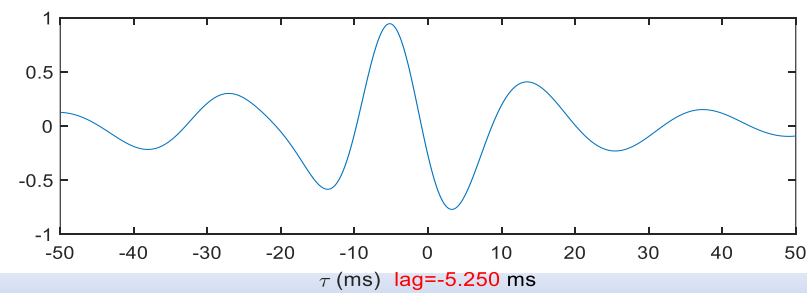
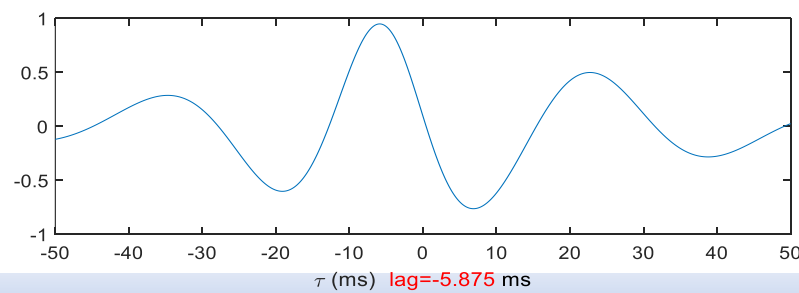
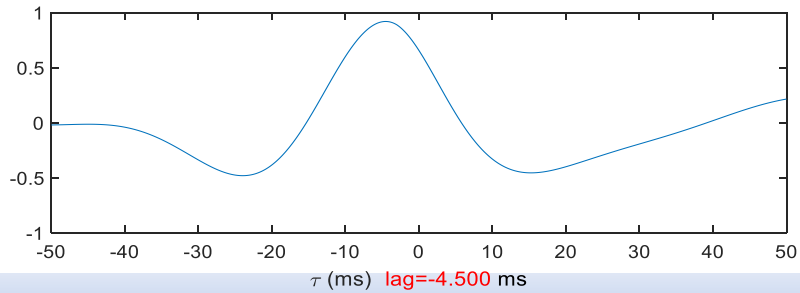
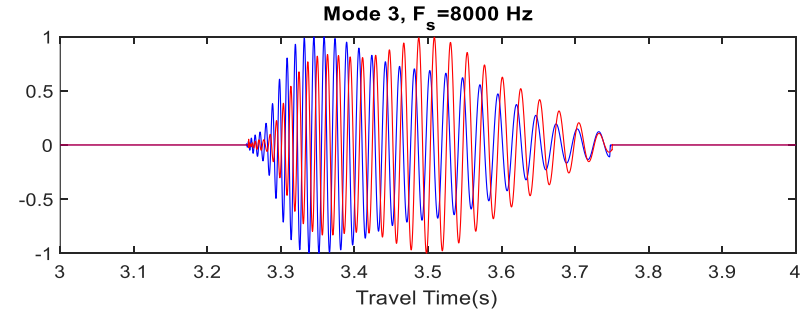
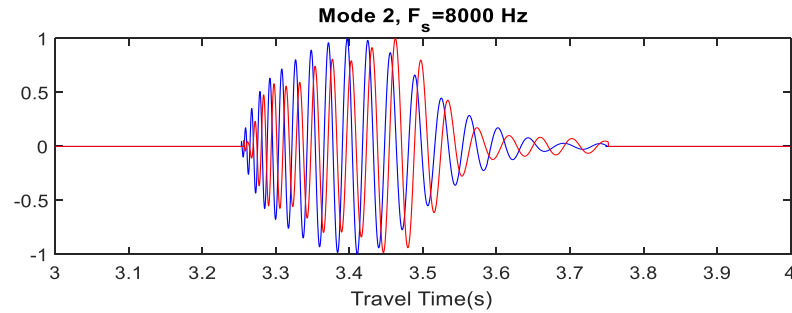
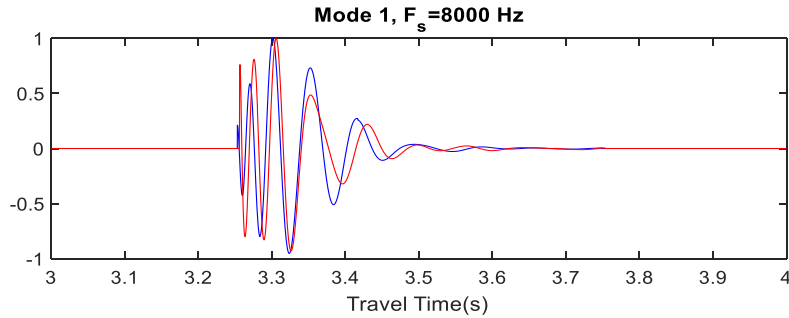
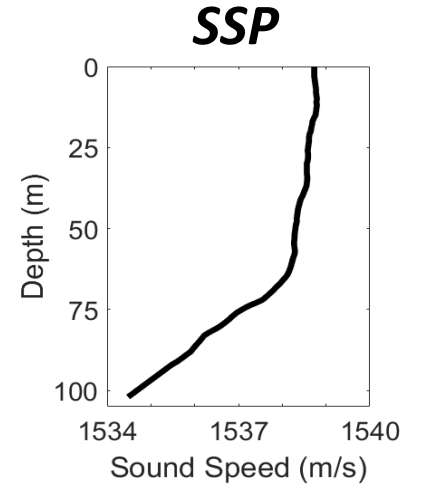
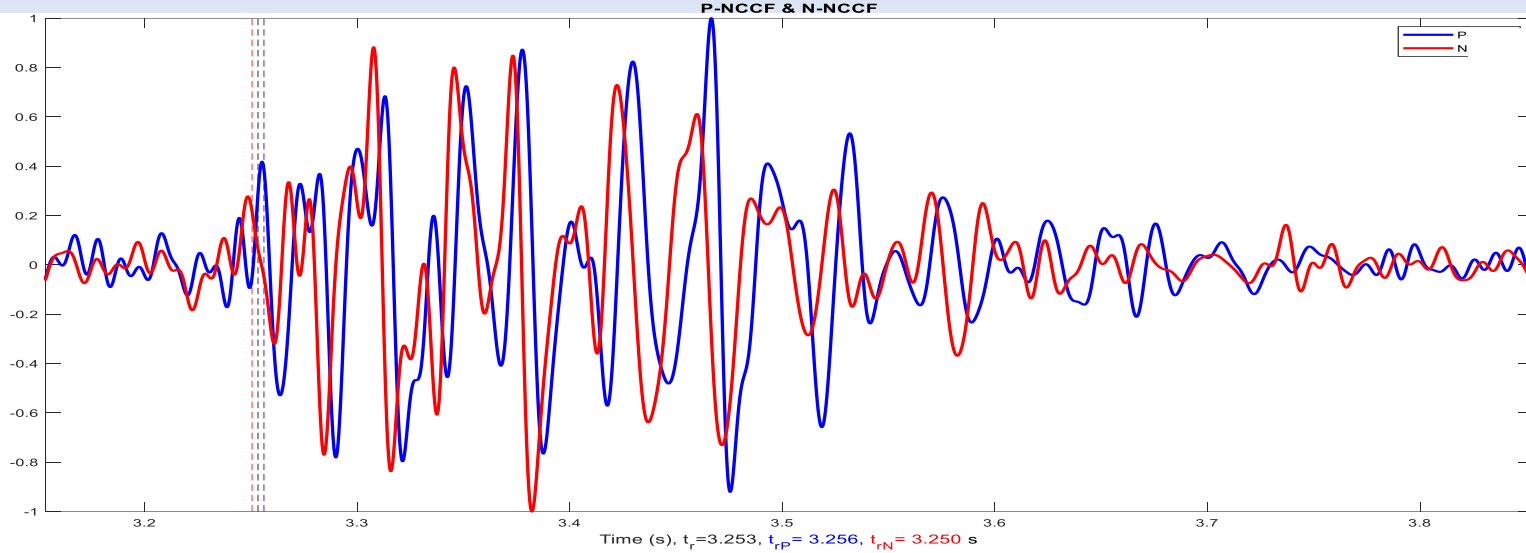
(a) The noise cross-correlation function  $C_{12}$  is shown for **P-NCCF** (solid line) and **N-NCCF** (dashed line) after removal of the relative drift of system clocks. The entire available data set is used for noise averaging. (b) Same as (a) but for  $C_{23}$ . (c) Correlation between the positive- and negative-time-delay parts of the cross-correlation functions  $C_{12}$  (solid line) and  $C_{23}$  (dashed line). The position of the peak of the correlation of correlations determines the nonreciprocity of travel times induced by currents at sound propagation between the respective pair of instruments.

Travel time nonreciprocity:  $\delta t = 2c^{-2}rU$ ,  $U = H^{-1} \int_0^H u_x(z) dz$  Measured current velocity:  $U_{12} = -0.47 \text{ m/s} \pm 7\%$ ,  $U_{23} = -0.49 \text{ m/s}$



O. A. Godin, M. G. Brown, N. A. Zabotin, L. Zabolina, and N. J. Williams, Passive acoustic measurement of **flow velocity** in the Straits of Florida, Geoscience Lett. 1, Art. 16 (2014)

# FL Future work: Passively quantifying current



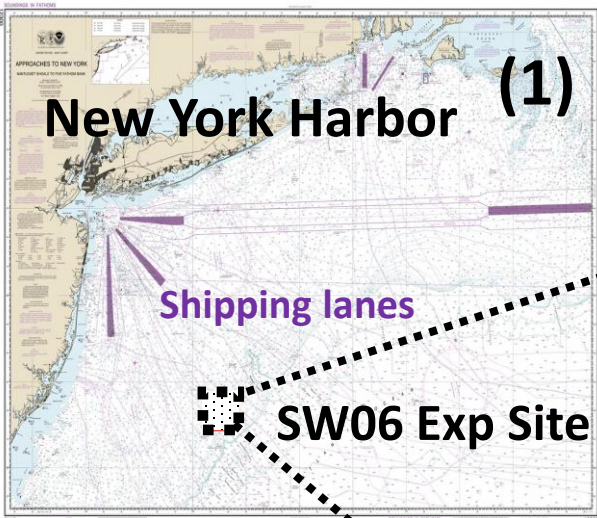
# Time Warping on Empirical Green's Functions (EGFs) from Shallow Water 2006 Experiment (SW06)



# Noise Cross Correlation Functions (NCCFs) : 15-day Avg



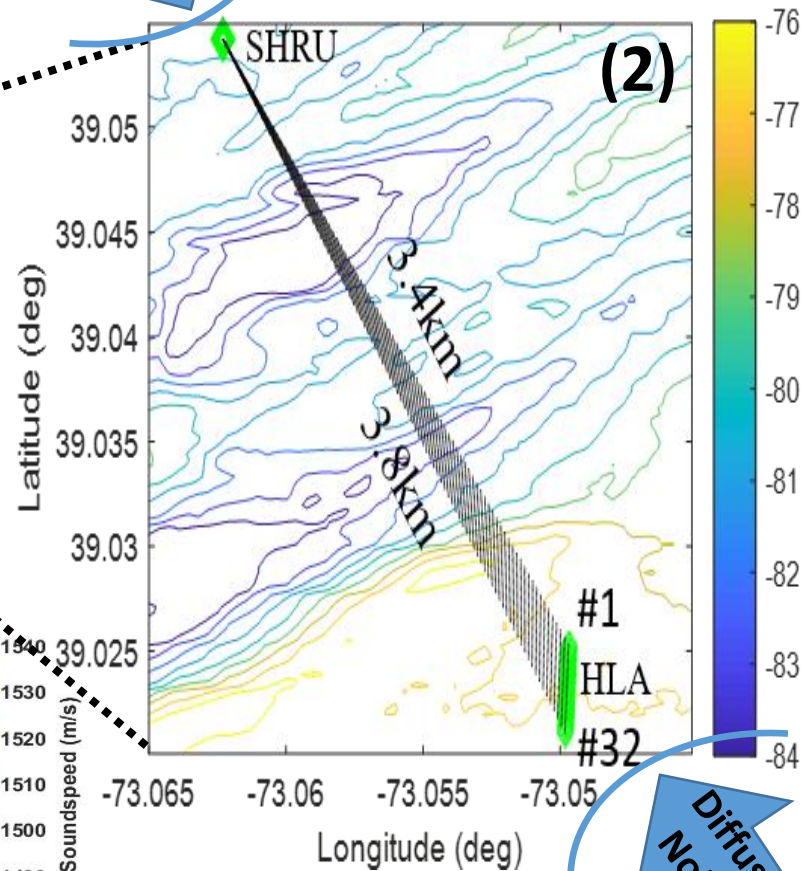
ROC Naval Academy



SW06 location in the chart "Approaches of New-York"

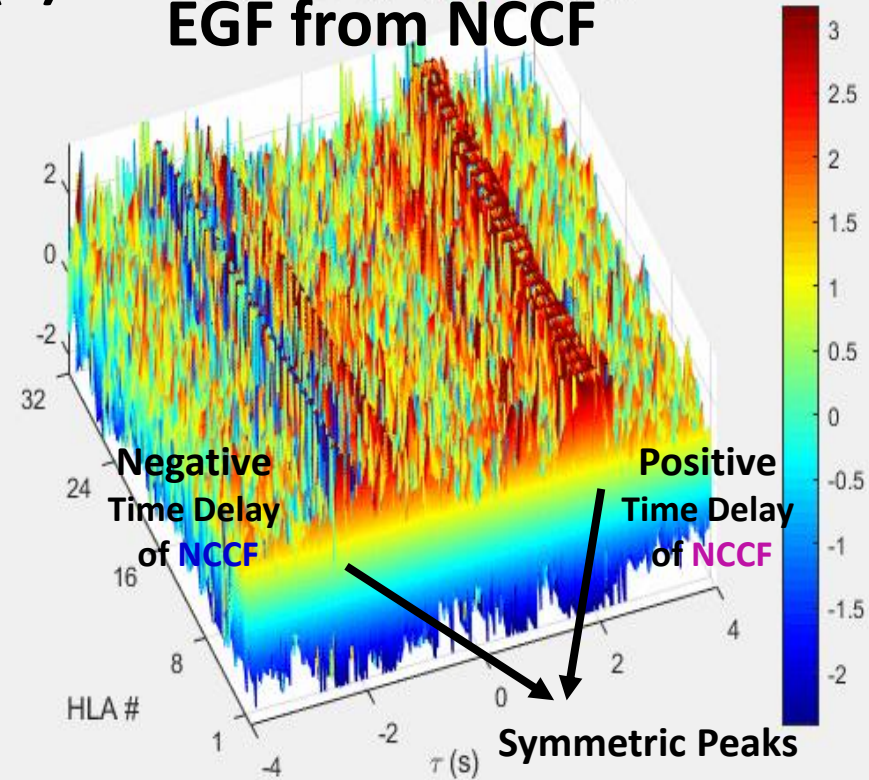
Diffuse Noise

## SW06 Exp



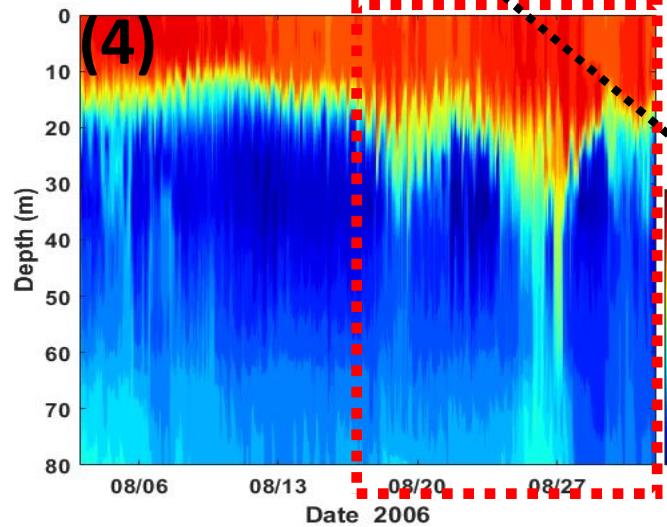
Diffuse Noise

### (3) NCCF, Averaging 01 Day(s) EGF from NCCF



$$\hat{C}_i(f) = \frac{1}{N} \sum_{n=1}^N \frac{P_{SHRU}^{(n)}(f) P_{HLA,i}^{(n)}(f)^*}{|P_{SHRU}^{(n)}(f) P_{HLA,i}^{(n)}(f)^*|}, \quad i = 1, 2, \dots, 32$$

$$\frac{d}{dt} C_i(t) = D(t) * [G(\mathbf{r}_{HLA,i}, \mathbf{r}_{SHRU}, -t) - G(\mathbf{r}_{SHRU}, \mathbf{r}_{HLA,i}, t)]$$



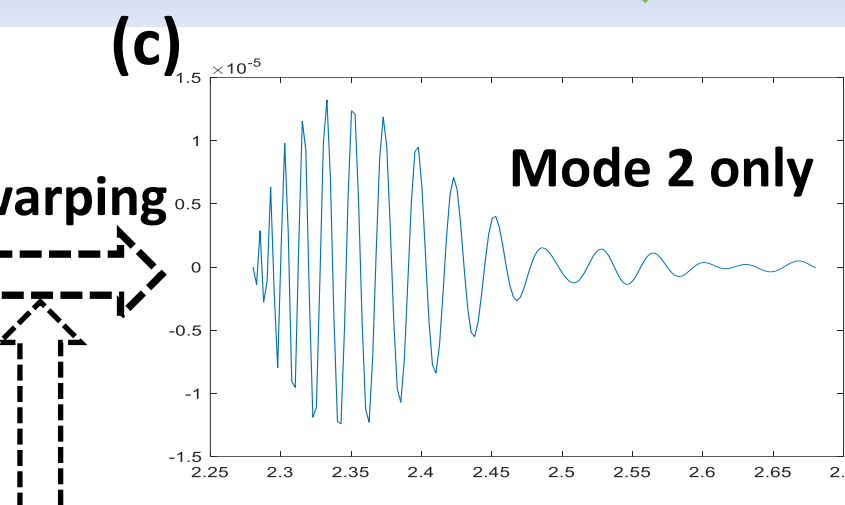
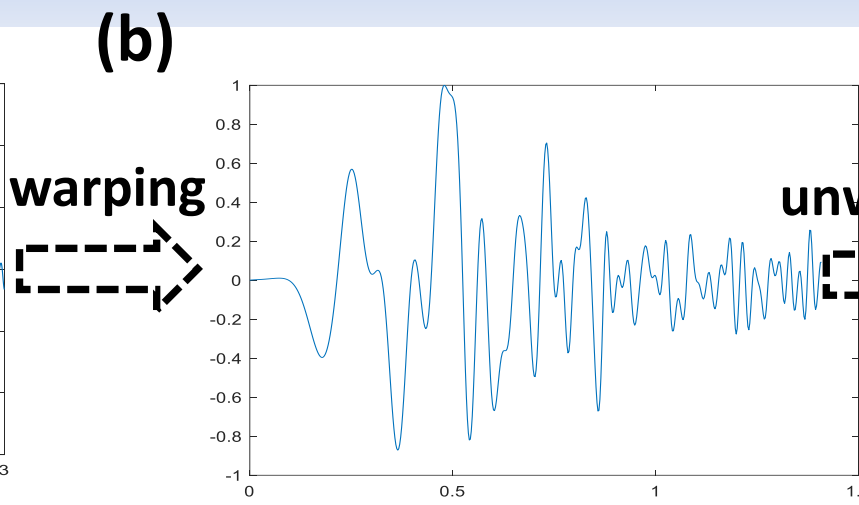
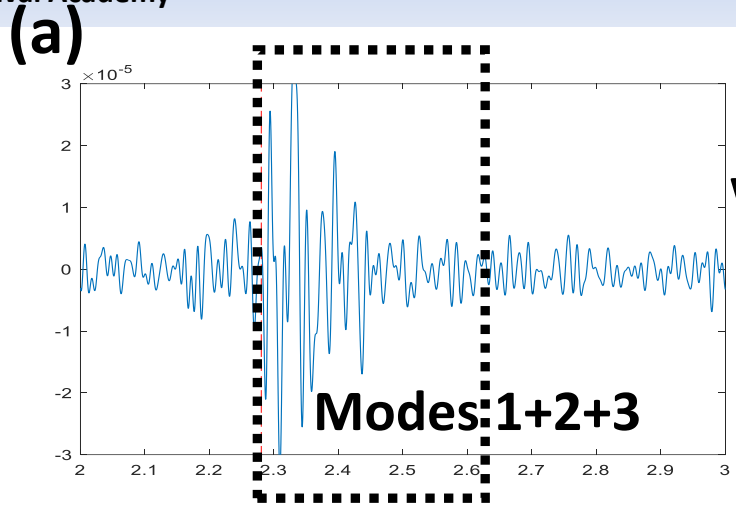




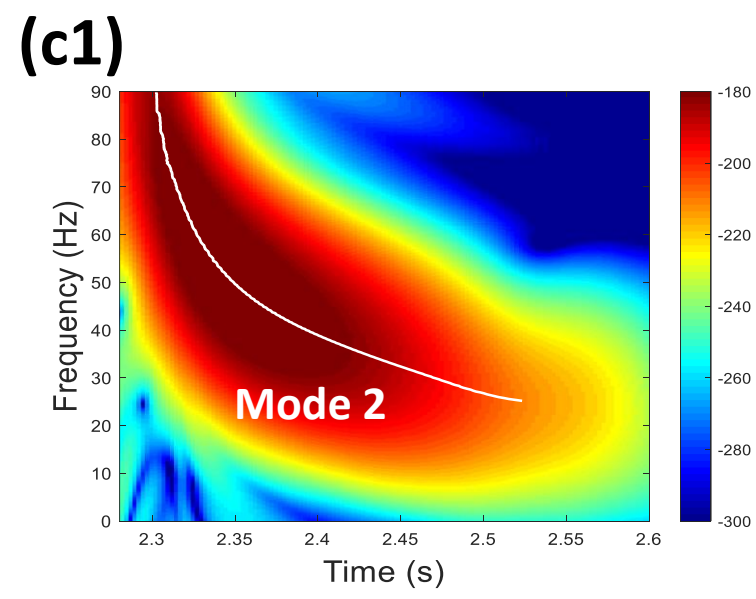
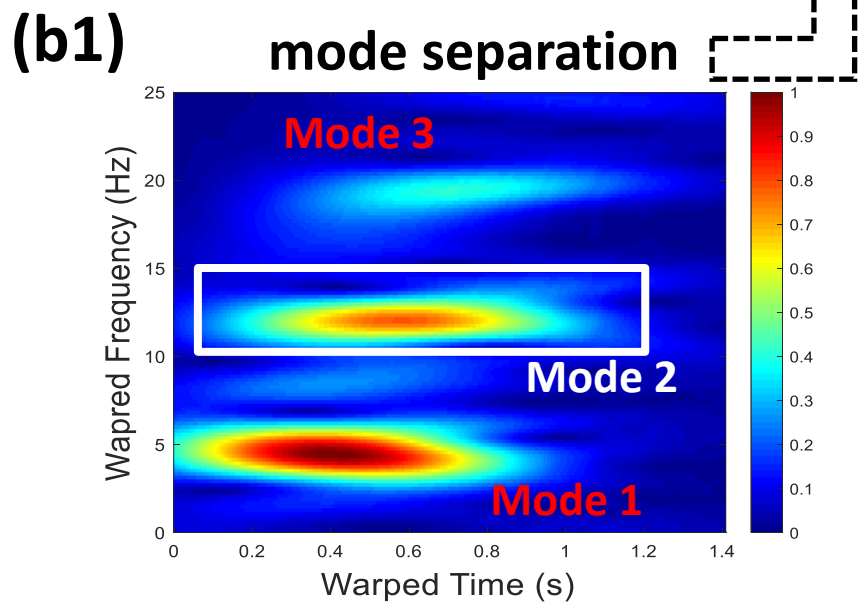
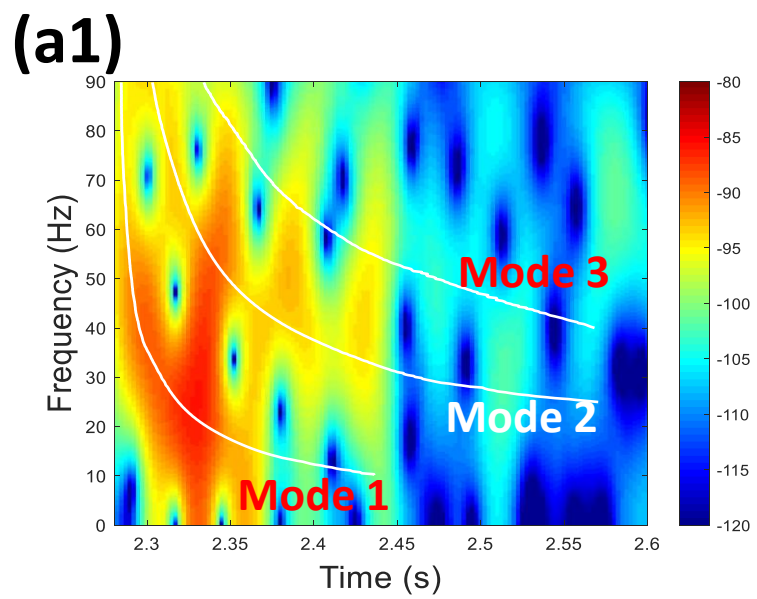
# Time Warping on NCCF (15-day avg)



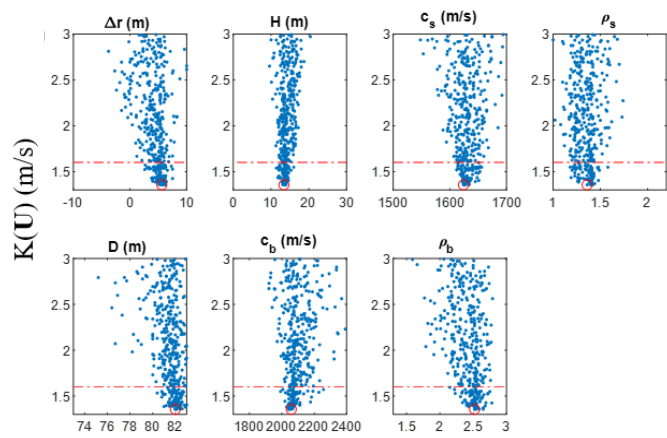
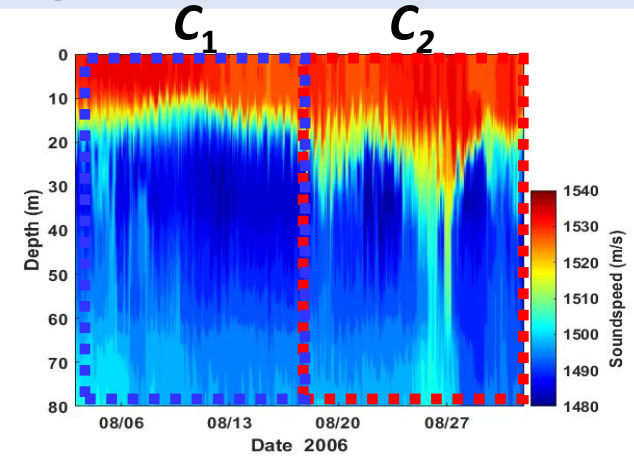
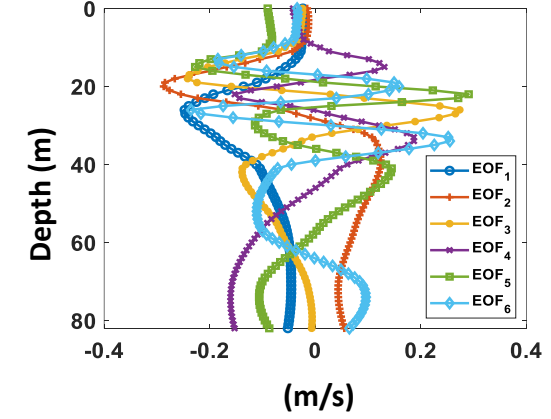
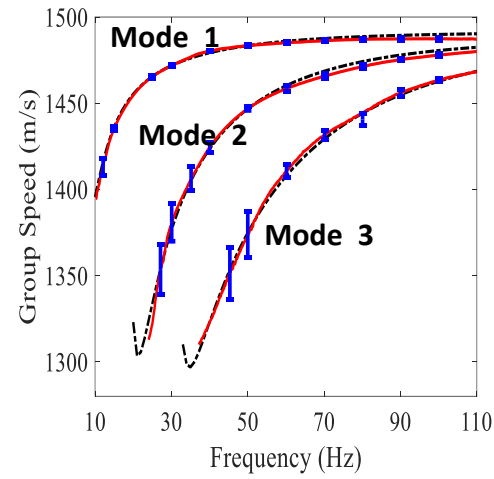
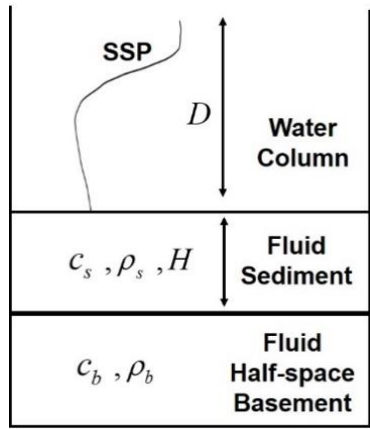
Signal



Spectrogram

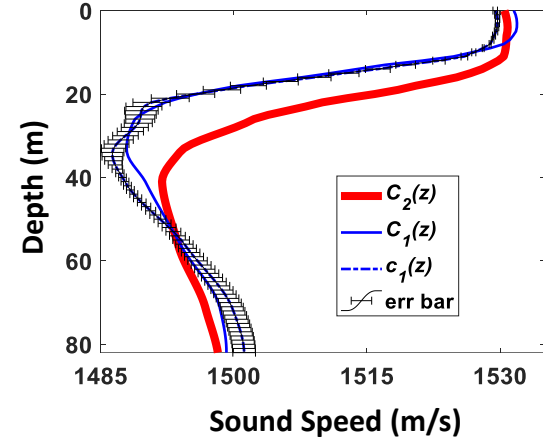


# Ocean Characterization by EGFs of Sub-Seasonal Sound Speed Variations : 15-day Avg

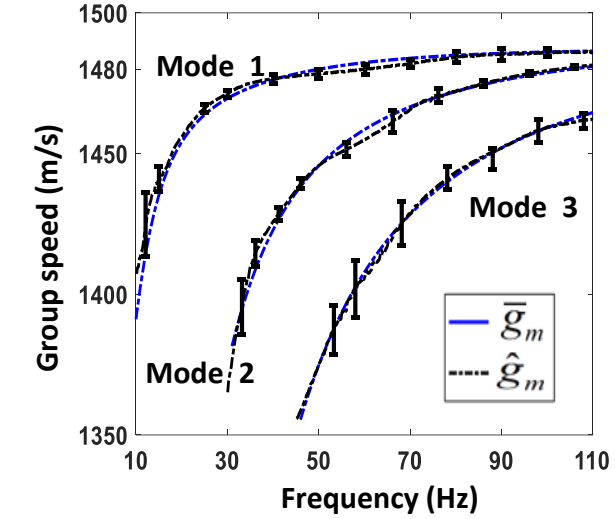


$$K(\mathbf{U}) = \sqrt{\sum_{m=1}^M \frac{1}{M N_m} \sum_{n=1}^{N_m} [\bar{g}_m(f_n) - \hat{g}_m(f_n, \mathbf{U})]^2}$$

**Geoacoustic Inv Seabed**



**SSP Inv  $c_1 \approx C_1$**

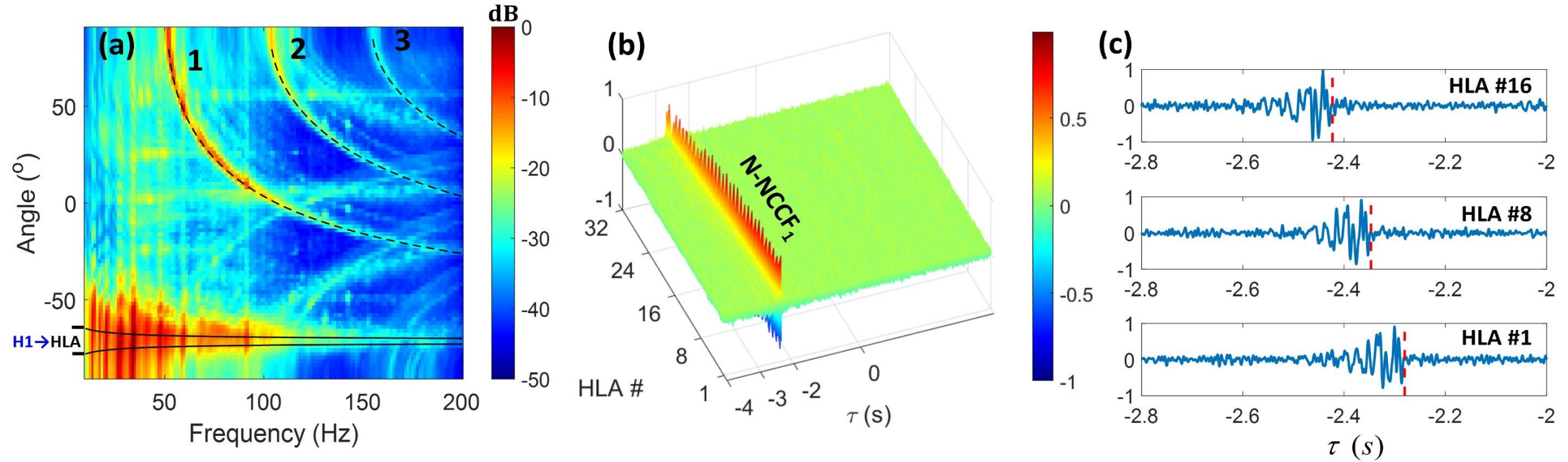


T. Tan, O.A. Godin, B.G. Katsnelson, and M. Yarina (2020), "Passive **geoacoustic inversion** in a dynamic environment on a continental shelf," *J. Acoust. Soc. Am.* 147, EL453-459

T. Tan and O. A. Godin (2021), "Passive acoustic characterization of **sub-seasonal sound speed variations** in a coastal ocean," *J. Acoust. Soc. Am.* 150.4 2717-2737.

$$w_m(\varphi) = e^{-ikd_m \sin(\varphi)}$$

$$B(\varphi_s) = \mathbf{w}^T \mathbf{p} (\mathbf{w} \mathbf{p})^\dagger = \mathbf{w}^T(\varphi_s) \mathbf{K}(\varphi) \mathbf{w}(\varphi_s)$$

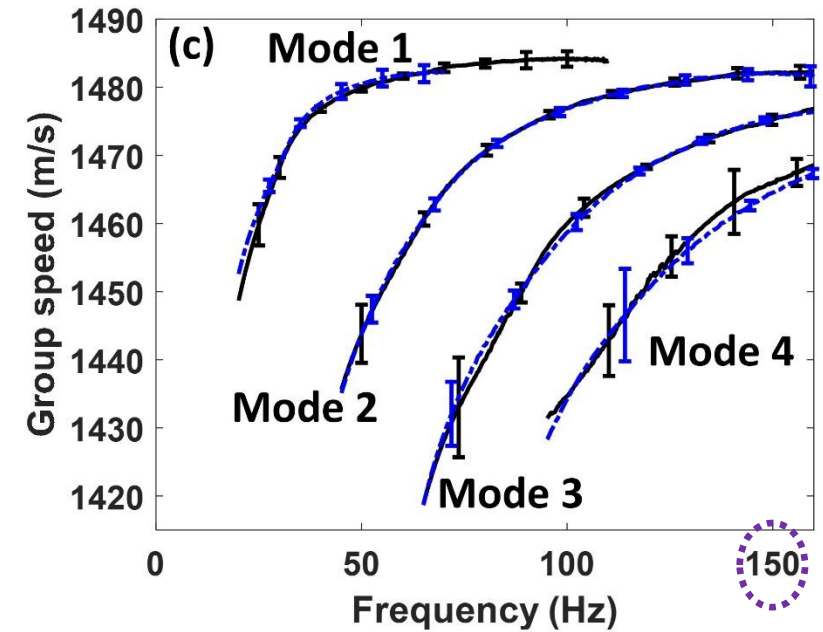
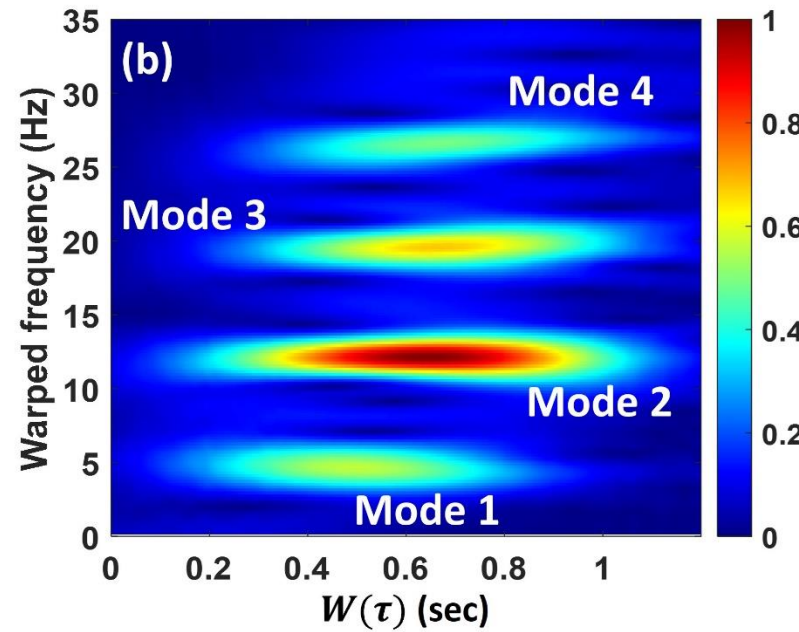
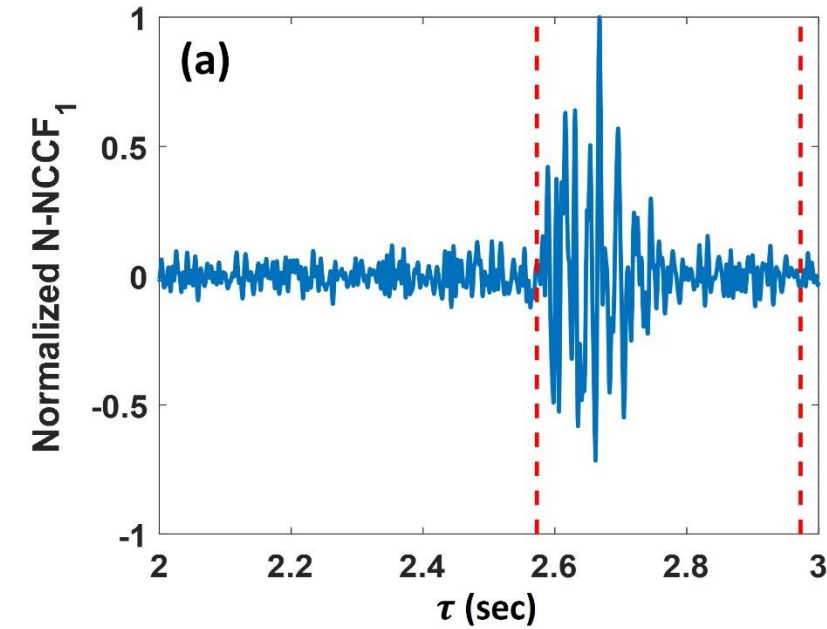


**Beamforming on HLA**

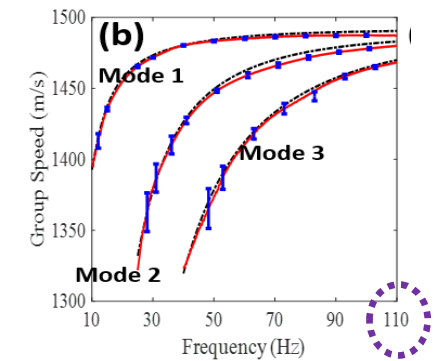
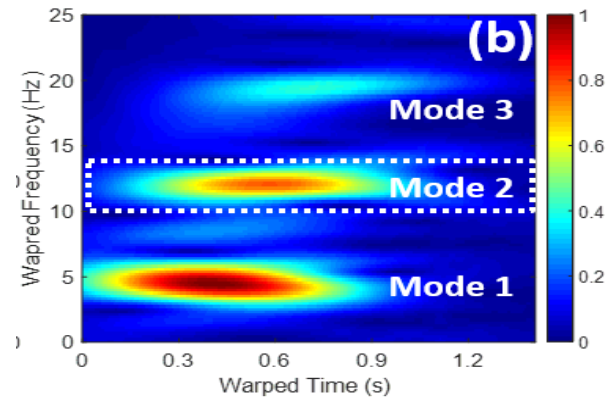
# Rapid N-NCCF: Broader Frequency

## Extra Higher Mode

## Broader Frequency Band



## Previous 15-day NCCF



# Conclusion on Time Warping in Underwater Acoustics Applications

- **Time Warping** suitable to analyze the **modal components** of **low-frequency** ( $<500$  Hz) acoustic Green's functions in **shallow water** (water depth  $<200$  m) after propagation **several kilometers** ( $>1$  km)
- **Applications: Geoacoustic Inversion, Source Localizations, Marine Mammals Vocalizations, Tomography, Vector sensor, Noise Cross-Correlation Function....**

**Thank You!**  
**Q & A**