# Comment on "Entropy, energy, and proximity to criticality in global earthquake populations" by Ian G. Main and Fahad H. Al-Kindy

Chien-chih Chen and Chun-Ling Chang

Department of Earth Sciences and Graduate Institute of Geophysics, National Central University, Chungli, Taiwan

Received 6 November 2003; revised 28 January 2004; accepted 14 February 2004; published 19 March 2004.

*INDEX TERMS:* 3220 Mathematical Geophysics: Nonlinear dynamics; 7209 Seismology: Earthquake dynamics and mechanics; 7223 Seismology: Seismic hazard assessment and prediction; 7260 Seismology: Theory and modeling. **Citation:** Chen, C.-c., and C.-L. Chang (2004), Comment on ''Entropy, energy, and proximity to criticality in global earthquake populations'' by Ian G. Main and Fahad H. Al-Kindy, *Geophys. Res. Lett.*, *31*, L06608, doi:10.1029/2003GL019008.

[1] It has been often suggested that earthquakes are somewhat similar to critical points of the Earth's brittle crust from a more global point of view [*Jaume and Sykes*, 1999]. Also a very universal, but quite ill-defined, perspective is proposed with the concept of self-organized criticality in the Earth Science community [*Bak and Tang*, 1989]. These various concepts from statistical physics probably model different properties of the Earth's crust at different time scales [*Huang et al.*, 1998], and whether criticality and self-organized criticality are compatible remains an open puzzle.

[2] The recent GRL contribution *Main and Al-Kindy* [2002] examines the question of proximity of the global earthquake population to the critical point characterized by fluctuations in the energy $E$ and entropy $S$ based on the Harvard CMT catalogue. Some macroscopic state variables from classical statistical physics, such as $S$, $\langle E \rangle$ and $\langle \ln E \rangle$, were calculated for global earthquakes, and the results are compared with a theoretical model corresponding to a Boltzmann probability density distribution of the energy with the form of $p(E) \propto E^{-B-1}e^{-E/\theta}$.

[3] The significance of *Main and Al-Kindy* [2002] may be twofold. First, they demonstrate that global seismicity is in a near-critical state with large fluctuations in mean energy but relatively small changes in entropy. This fact highlights the need for calculation of $S$ as well as $E$ when investigating any sort of complex dynamical system in the context of criticality and/or self-organized criticality. This also initiates their major concern: is the fluctuation of ±10% in entropy allowed in ideal self-organized critical system? Inspired by this result, soon after the publication of *Main and Al-Kindy* [2002], *Goltz and Böse* [2002] concluded that the Earth's crust is in a state of intermittent caiticality, supported by the significant fluctuations of ±20% in configurational entropy.

[4] Secondly, some kind of non-equilibrium systems, such as slider block models or lattice threshold models, fluctuate around a time-averaged steady state and, in a simplistic sense, are similar to equilibrium systems that also fluctuate around the time-averaged equilibrium state [*Rundle et al.*, 1997]. If it could be shown that broad classes of these non-equilibrium models were in fact isomorphic to equilibrium model, an extremely powerful set of methods and results for equilibrium systems would be immediately available. In effect, the answer to this issue depends on understanding whether these non-equilibrium systems obey the postulate of equal a priori probability. *Rundle et al.* [1995] have found that broad classes of lattice models possess an energy distribution approaching a Boltzmann distribution. Now, after the work of *Main and Al-Kindy* [2002], it seems true that statistical mechanical concepts from equilibrium thermodynamics can apply in some circumstances to far-from equilibrium threshold systems. Such belief displayed in *Main and Al-Kindy* [2002] comes from the apparently successful, but in fact false, prediction of $B$-value (see explanation below) from the crucial relationship between energy and entropy, i.e., equation (5) in *Main and Al-Kindy* [2002] or equation (1) in the following.

[5] In the Appendix of *Main and Al-Kindy* [2002], for a system in a near-critical state, they derived from the standard strategy of equilibrium thermodynamics the following relationship

$$S = S_0 + B\langle \ln E \rangle. \qquad (1)$$

Then, by plotting the calculated points of $S$ and $\langle \ln E \rangle$, the $B$-value for global earthquakes was inferred from the slope of the best-fit regression line for equation (1) (see Figure 2 in *Main and Al-Kindy* [2002]). This result agrees very well with the independent maximum likelihood estimations, previously reported by *Kagan* [1999] and *Leonard et al.* [2001], for the same Harvard CMT catalogue. However, due to one critical mistake, equation (1) is actually incorrect. The critical mistake made in *Main and Al-Kindy* [2002] was the neglect of $E$ in the denominator of the logarithmic term in the kernel of their equation (A5). Therefore, for a system in a near-critical state, the correct form of the relationship between energy and entropy should be

$$S = S_0 + (B+1)\langle \ln E \rangle. \qquad (2)$$

[6] Consequently, the prediction of the $B$-value for global earthquakes from the method of *Main and Al-Kindy* [2002] fails, and the developed strategy from equilibrium thermodynamics of attacking far-from equilibrium threshold systems seems no longer as promising as demonstrated in *Main and Al-Kindy* [2002].


0094-8276/04/2003GL019008$05.00

## References

Bak, P., and C. Tang (1989), Earthquakes as a self-organized critical phenomenon, *J. Geophys. Res.*, *94*, 15,635–15,637.

Goltz, C., and M. Böse (2002), Configurational entropy of critical earthquake populations, *Geophys. Res. Lett.*, *29*(20), 1990, doi:10.1029/2002GL015540.

Huang, Y., H. Saleur, C. G. Sammis, and D. Sornette (1998), Precursors, aftershocks, criticality and self-organized criticality, *Europhys. Lett.*, *41*, 43–48.

Jaume, S. C., and L. R. Sykes (1999), Evolving towards a critical point: A review of accelerating seismic moment/energy release prior to large and great earthquakes, *Pure Appl. Geophys.*, *155*, 279–305.

Kagan, Y. Y. (1999), Universality of the seismic moment-frequency relation, *Pure Appl. Geophys.*, *155*, 537–573.

Leonard, T., O. Papasouliotis, and I. G. Main (2001), A Poisson model for identifying characteristic size effects in frequency data: Application to frequency-size distributions for global earthquakes, ‘‘starquakes’’, and fault lengths, *J. Geophys. Res.*, *106*, 13,473–13,484.

Main, I. G., and F. H. Al-Kindy (2002), Entropy, energy, and proximity to criticality in global earthquake populations, *Geophys. Res. Lett.*, *29*(7), 1121, doi:10.1029/2001GL014078.

Rundle, J. B., W. Klein, S. Gross, and D. L. Turcotte (1995), Boltzmann fluctuations in numerical simulations of nonequilibrium lattice threshold systems, *Phys. Rev. Lett.*, *75*, 1658–1661.

Rundle, J. B., S. Gross, W. Klein, C. Ferguson, and D. L. Turcotte (1997), The statistical mechanics of earthquakes, *Tectonophysics*, *277*, 147–164.

C.-L. Chang and C.-c. Chen, Institute of Geophysics, National Central University, No. 300, Jung-da Road, Chungli 320, Taiwan. (s123@sal.gep.ncu.edu.tw)