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FAST TRACK PAPER

The fractal geometry of the surface ruptures of the 1999 Chi-Chi earthquake, Taiwan

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SUMMARY

The 1999 September 20 Chi-Chi earthquake is the largest seismic event which occurred in the island during the twentieth century. Available seismic data relative to this earthquake are of high quality, and surface ruptures identified as features associated to the Chelungpu fault can be clearly observed at the surface and precisely mapped. We calculated the fractal dimension (D) and b value of Gutenberg–Richter law for 6-month aftershocks of the Chi-Chi earthquake for the fault area, and find that the surface ruptures exhibit self-similar geometry only within specific ruler intervals. The D values of the surface ruptures reflect the fault slip and geometry at depth. More importantly, the small-size aftershocks seem more likely to occur within high D value and high b value areas, whereas small D value and small b value areas have a high potential for medium- and large-size aftershocks.

Key words: b value, fractal dimension, rupture.

1 INTRODUCTION

The Chi-Chi earthquake occurred in the central Taiwan on 1999 September 20 with local magnitude $\mathbf{M}_\mathrm{L} = 7.3$. This earthquake induced extensive surface ruptures along the Western Foothills of Taiwan, which extend for more than 100 km along a roughly N–S trend from the Chushan town (Nantou country) in the south to the Jhuolan town of the Miaoli country in the north (CGS 1999a,b; Shin 2000; Lin *et al.* 2001; Shin & Teng 2001). The high quality seismological network of Taiwan (Shin 2000; Shin & Teng 2001) has recorded available seismic data relative to this earthquake, whereas crustal deformation associated with the same event was detected by the Taiwan Global Positioning System (GPS) network (Yu *et al.* 2001a). Soon after the earthquake, the Central Geological Survey of Taiwan mapped the surface ruptures and identified them as the surface expression of the Chelungpu fault (CGS 1999a,b) (Fig. 1).

As it is well known that earthquake properties are closely related to the irregularities on a seismogenic fault, it also follows that an appropriate knowledge of the detail of the fault geometry (i.e. of the fault-related surface features) is fundamental for a better understanding of fault mechanics (Okubo & Aki 1987). To this aim, fractal analysis may play an important role for describing complex natural features and phenomena (Mandelbrot 1983; Turcotte 1989). Okubo & Aki (1987), for example, used fractal analysis to measure fault complexity and fragmentation along the San Andreas fault system, in central and southern California. With this approach, they anticipated a few relevant characteristics of the faulting process, and related them to the fractal geometry of the fault traces. According to Aviles & Scholz (1987), however, the fractal dimension (D) of the fault traces along the San Andreas shows that the fault surface is quite simple and near-planar; consequently they claim that creep, seismic slip and microearthquake do not display any correlation with the D value of the analysed fault segments. On the other hand, the spatial pattern of the northern Egyptian fault system and the fault plane solution of the 1992 October 12 Cairo earthquake, allowed Arab *et al.* (1994) to assess that faults with small D values are the most active. They also concluded that the fractal geometry of fault systems is a powerful tool for assessing risks related to specific faults in seismic active areas. The discontinuity patterns and variations of the D values of the Sumatra active fault system were used by Sukmono *et al.* (1996) to estimate the characteristics of the geodynamic processes in the area. The spatial and temporal distributions of earthquakes in Sumatra appear in fact to be strongly related to the D values of the fault system, and the recurrence interval of the D values in the segment classes may be used to predict future large earthquakes (Sukmono *et al.* 1997). Cello (1997) measured the D values of the Quaternary fault system in the central Apennines (Italy) and suggested that faults within the system are characterized by two D values (1 and 1.6). Based on this observation, the author concluded that the analysed system behaves as a young immature fault structure where linking segments generate medium-size earthquakes (Cello 2000).

In the case of the surface ruptures of the Chi-Chi earthquake, these have been clearly identified (CGS 1999a) or detected (Wang 2002;

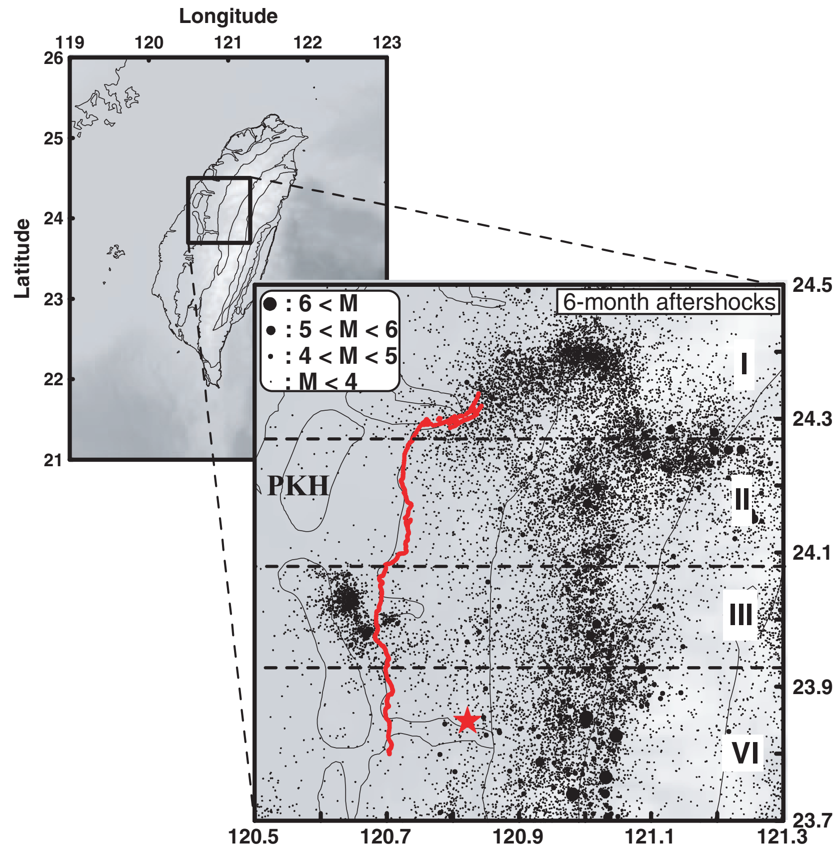

Figure 1. Red line is the surface ruptures along the Chelungpu fault zone; the red solid star represents the epicenter of the Chi-Chi earthquake, and the 6-month aftershock distribution of the Chi-Chi earthquake. The Chelungpu fault is divided into four segments which are labelled as I, II, III and IV from north to south.

Wang *et al.* 2002), and appear to be distributed in a zone of several tens of metres wide (Lin *et al.* 2001). Since the seismic, geodetic and geologic data available for the area are all high-quality, the Chi-Chi earthquake provides an excellent opportunity to assess whether or not these newly generated coseismic surface ruptures display fractal properties. We also want to investigate whether the fractal geometry of the coseismic surface ruptures are in some way related to the fault slip, fault geometry and to the nature and distribution of the aftershocks. Finally, we wish to evaluate whether and how fractal analysis may be used for assessing risks related to faulting in seismic active areas.

2 FRACTAL ANALYSIS

Traditionally, Euclidean geometry has served as the basis of the intuitive description of the geometry of nature. However, many objects in nature are so complicated and irregular that it is difficult to model or describe them by the traditional Euclidean geometry. In the 1970s, Mandelbrot (1967) developed a heuristic method to describe complex forms in nature. Fractal analysis provides a measure of complexity by determining a D value (the fractal dimension) which determines the relative importance of large versus small scale features. This concept is successfully and widely used in various fields, for example, in physics, biology and earth science (Mandelbrot 1983).

In this study, we derive D values to express the complexity of the surface ruptures of the Chi-Chi earthquake. We adopt the Okubo & Aki's (1987) method, which uses a minimum number $[N(r)]$ of circles with a chosen radius (r) to cover the fault-related surface ruptures. The fault length $[L^{\circ}(r)]$ is normalized by a factor of $\pi/4$ and is a function of radius r. That is, $L^{\circ}(r) = 2N(r)r$. If we plot $L^{\circ}(r)$ as a function of measuring radius r on log–log axes, then the D value can be estimated, from the slope (c) of the best-fitting straightline to the data set, as $D = 1 - c$. When D is large (i.e. $D > 1$) it means that the geometry of the surface rupture is quite irregular, whereas $D = 1$ describes a straightline rupture.

For this study, we used the map of the surface ruptures of the Chi-Chi earthquake at the scale of 1:25 000 compiled by the Central Geological Survey of Taiwan (CGS 1999b). The lengths of the ruptures were measured used 11 circles whose radii (r) are from 0.1 km (inner cut-off) to 1 km (outer cut-off). According to the characteristics of the coseismic displacement, geometry of the surface rupture zone and geological structures, Lin *et al.* (2001) divided the Chelungpu surface rupture zone into four segments (I–IV) extending from north to south (Fig. 1). In Fig. 2, the fault length (L°) and the radius (r) of each segment are plotted on log–log axes.

The fault length appears self-similar only when the radius of the circle is less than certain characteristic length (r_c). As may be seen in Fig. 2, the r_c value for segment I is 0.8 km whereas for the other segments r_c is larger than 1 km. Radii which are close to the unit length (r_0) of the ruptures on the map show that the fault length has a constant value in the left portion of the diagram (i.e. at small scale values). This implies that the D values computed from the 1:25 000 scale map of the surface ruptures of the Chi-Chi earthquake cannot be resolved for radii that are less than 0.2 km. In Table 1, the D values for each fault segment, the correlation coefficient (R), r_0 and

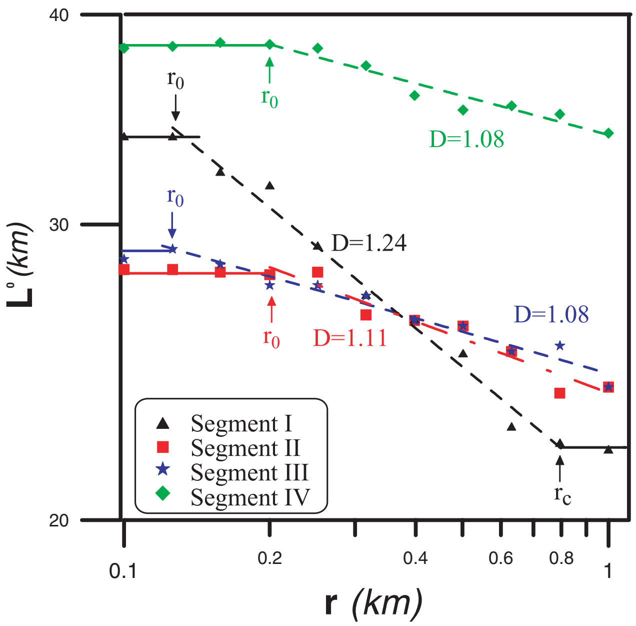

Figure 2. Log–log diagram of fault length vs. measuring radius for the four segments of the Chelungpu fault.

r_c estimated from the slopes of straightline least squares fitting are all listed. As shown, the D value for the total length of the surface ruptures is $1.12 + 0.014$ (*SD*), whereas from the north to the south, the D values range from 1.24 to 1.02. The largest D value is at the northern segment (I), while the southern segments (III and IV) have the same D values.

3 DISCUSSION

Due to the range of the fault area selected for counting, the D value of a fault system may be quite different. Okubo & Aki (1987) for example, analysed the fault traces along a 30-km-wide fault zone of the San Andreas system. They found a D value of 1.3. However, as mentioned above, Aviles & Scholz (1987) derived a D value of about one for the main fault traces exposed in the same area. In this study, almost all the surface ruptures of the Chi-Chi earthquake are distributed in a deformation zone of several meters wide and they can be clearly and directly observed in the field. This assures that all the ruptures can be accurately charactered by their fractal dimension (i.e. by their D values).

The nature of the fault trace patterns at the surface records their time–space evolution, as well as fault activity during a (or several) geological time period(s). Furthermore, their fractal properties suggest that the D value of a newly formed fault system evolves from a non-fractal ($D = 1$) to a fractal ($1 < D < 2$) geometry (Cello 1997). The surface ruptures of the Chi-Chi earthquake are due to a single seismic event, and they can, therefore, be considered as an instantaneous response to the applied tectonic stress. The geometric complexity and fragmentation pattern of the surface ruptures are expressed by self-similar (or scale invariant) geometric features in a specific ruler interval (r_0–r_c). When the ruler is greater than a certain characteristic length, r_c, the fractal geometry of the ruptures will loose self-similarity. If the ruler is less than unit (r_0) of the map, the fractal geometry of the ruptures cannot be resolved from the adopted rupture map.

The coseismic ground displacement of the Chi-Chi earthquake, obtained from GPS data (CGS 1999a; Yu *et al.* 2001a), strong motion seismogram (Shin 2000; Shin *et al.* 2001), and field measurement (CGS 1999a; Lin *et al.* 2001), suggest that earthquake deformation occurred mostly in the fault hanging wall (Fig. 3a). Available data also show that the coseismic ground displacement increases from south (2 m) to north (9 m) along the fault zone, and those areas of large coseismic strain are typically faulted up to the surface. Consequently, segments with large coseismic ground displacement display higher D values.

Fault slip, geometry, and length of the Chelungpu fault were also inferred from teleseismic data (Lee & Ma 2000; Xu *et al.* 2002), near field strong motion data, and GPS data (Johnson *et al.* 2001; Ma *et al.* 2001; Wu *et al.* 2001; Johnson & Segall 2004); based on this information, we conclude that most of the slip is concentrated at shallow depth (i.e. less than 20 km) (Fig. 3b). Furthermore, we observe that the spatial distribution of the slip is generally consistent with field observations. In other word, slip increases towards the northern end of the Chelungpu fault, and its maximum value, recorded along the northern section of the fault, is about 20 m. Although Aviles & Scholz (1987) have shown that seismic slip is uncorrelated with the D values obtained for the San Andreas fault system, but our results indicate that fault slip is strongly related with the measured D values instead: the larger the fault slip, the higher D value is.

Some authors have also suggested that the geometric complexity of the Chelungpu fault cannot be resolved by assuming a single-plane fault model (see, e.g. Lee & Ma 2000; Xu *et al.* 2002); a multiplane fault model in fact fits better the observed data (Johnson *et al.* 2001; Ma *et al.* 2001; Wu *et al.* 2001; Johnson & Segall 2004). In particular, in order to model the northern-end of the Chelungpu fault structure, one needs to include at least one (or even two) fault splay. This would simulate better the complexity of the structure at its northern-end, compared with other segments of the Chelungpu fault. As concerns fault area data, unfortunately there is no direct method available to measure fault surfaces at depth. Aviles & Scholz (1987) proposed that the geometry of the surface fault traces may be used to infer the complexity of the fault structure at depth. Our results

Table 1. D values with their correlation coefficient (R), r_0 and r_c of the surface ruptures of the Chi-Chi earthquake, and b values of Gutenberg–Richter law for 6-month aftershocks of the Chi-Chi earthquake.

Segments	I	II	III	IV	Total
D	1.24 ± 0.016	1.11 ± 0.031	1.08 ± 0.016	1.08 ± 0.016	1.12 ± 0.014
R	0.9866	0.9357	0.9623	0.9306	0.9727
r_c (km)	0.8	>1	>1	>1	>1
r_0 (km)	0.126	0.2	0.126	0.2	0.126
b	1.14 ± 0.017	0.96 ± 0.030	1.01 ± 0.058	0.75 ± 0.036	0.93 ± 0.034
			0.84 ± 0.036		

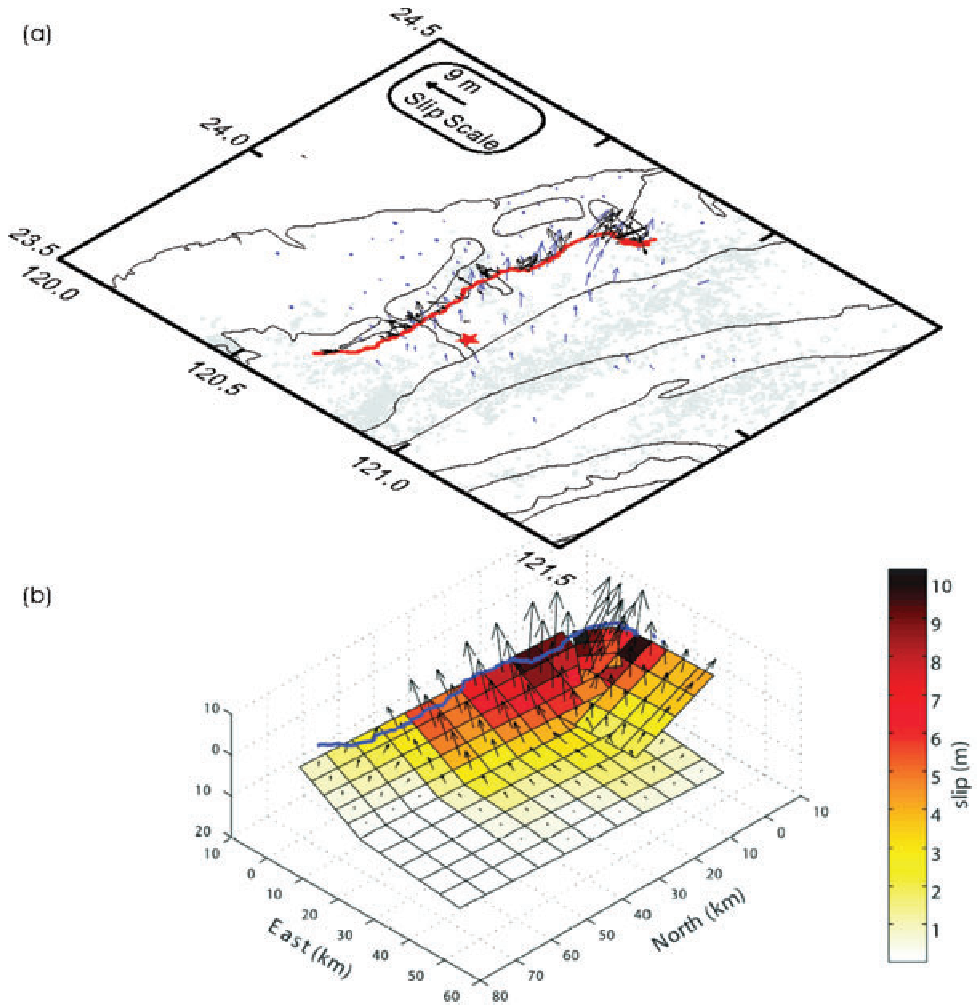

Figure 3. (a) Black arrows are the geological horizontal slip (CGS 1999a) and blue arrows are the GPS horizontal slip (Yu *et al.* 2001b) on the surface of the Chelungpu fault. (b) Fault geometry and slip distribution on faults (Johnson & Segall 2004).

confirm their statement that the northern part of the Chelungpu fault with more complex fault structure exhibits higher *D* value of the surface ruptures.

In order to collect additional information concerning the relationships between seismicity and the fractal geometry of the fault system, in the following section we illustrate briefly the main characteristics of seismicity in western central Taiwan. In this area, the seismicity surrounds the PeiKang Basement High (PKH) area (Wang & Shin 1998) (Fig. 1), and most of the aftershocks are distributed on the eastern side of the Chelungpu fault. There is a low aftershock activity along the surface rupture area (Chang *et al.* 2000; Wang *et al.* 2000). The *b* values of Gutenberg–Richter law for 6-month aftershocks of the Chi-Chi earthquake in each segment were calculated (shown in Table 1 and Fig. 4). The result shows that the *b* values appear strongly related to the *D* values. Based on 4-hr, 4-month (Figs 4 and 5 in Wang *et al.* 2000) and 6-month (Fig. 1) aftershock distributions of the Chi-Chi earthquake, we notice that the medium and large earthquakes seem more likely to occur in areas with small *D* values and small *b* values (segments II, III and IV), while small earthquakes (magnitude less than 4) look more likely to occur in areas with high *D* values and high *b* values (segment I). This observation imports an important notion that the fractal dimension of the surface ruptures is related to the *b* value of the aftershocks, and

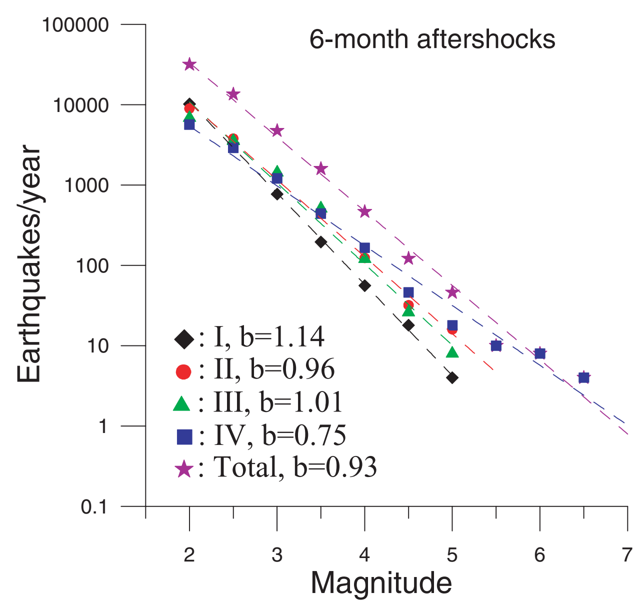

Figure 4. The *b*-values of Gutenberg–Richter law for 6-month aftershocks of the Chi-Chi earthquake for each segment.

suggests a direct and intrinsic connection between seismicity and fault structure.

4 CONCLUSIONS

The *D* values obtained for the surface ruptures associated with the Chi-Chi earthquake suggest that these features exhibit self-similar distributions within well-defined ruler intervals. The complexity of the surface ruptures results from the nature of the geological structure and from fault slip and geometry. In the case of the Chi-Chi earthquake, the *D* values of the surface ruptures relate well with the coseismic ground displacement, fault slip and the geometric complexity of the Chelungpu fault.

The earth surface at the northern end of the Chelungpu fault (showing high *D* values) appears to be composed of fragmented blocks, showing more complexity with respect to the southern end of the structure (characterized by small *D* values). The *b* values of the aftershocks show strongly related the *D* values. Small-size aftershocks (magnitude less than 4) are frequent in high *D* value and high *b* value areas, whereas medium and large-size aftershocks often take place at small *D* value and small *b* value areas. Therefore, the *D* value of the surface ruptures of a large earthquake may probably be used for assessing the seismic risk associated with aftershock sequences.

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