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Collective Weibull behavior of social atoms: Application of the rank-ordering statistics to historical extreme events

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Abstract – Analogous to crustal earthquakes in natural fault systems, we here consider the dynasty collapses as extreme events in human society. Duration data of ancient Chinese and Egyptian dynasties provides a good chance of exploring the collective behavior of the so-called social atoms. By means of the rank-ordering statistics, we demonstrate that the duration data of those ancient dynasties could be described with good accuracy by the Weibull distribution. It is thus amazing that the distribution of time to failure of human society, *i.e.* the disorder of a historical dynasty, follows the widely accepted Weibull process as natural material fails.

Introduction. – Physicists have long tried to apply their skills to fields outside of physics and, over the last few decades, the interdisciplinary fields of research including *Sociophysics* and *Econophysics* have been thrivingly grown (*e.g.*, [1–6]). Both the Sociophysics and Econophysics are intended as the interdisciplinary research fields applying theories and methods mainly developed by statistical physicists to solve problems in society and economics. One of the landmarks in scientific philosophy obtained from numerical simulations in these interdisciplinary fields is that plain macroscopic behavior could emerge from the complicated microscopic interactions between a vast amount of agents (*e.g.*, [3,5,7]). Together with numerical simulations, the techniques based on observations of data mining and pattern recognition could help explain the underlying process behind the data and thus gain insight into the nature of investigated phenomena. For example, the statistics of extreme events has been important to focus on the study of complex systems (*e.g.*, [4,8,9]). Extreme stock market fluctuations often result in large financial losses; earthquakes and floods can kill thousands of people; and global terrorism is strongly linked to political extremism.

Statistical distributions of extreme events in the stock market and various natural systems have been pervasively investigated for several decades (*e.g.*, [8–14]), whereas the studies of political/social extreme events are rarely presented in the literature (*e.g.*, [15]). The rarity for the study of extreme events in Sociophysics might be due to the ambiguity in the definition of social extreme events [16,17], together with inaccessibility of historical data in the society of human beings. Helbing *et al.* proposed disasters as extreme events in the sense of the amounts of victims and economic losses [16]. Thus, the collapse of an existed dynasty is undoubtedly a typical example of extreme events in Sociophysics. It therefore seems interesting and fundamentally important to analyze the historical data of dynasties by means of the sophisticated statistics of extreme events (*e.g.*, [4,8,18]). Waldrop started his book with several thoughtful questions [1]. One of them is "*Why did the Soviet Union's forty-year hegemony over Eastern Europe collapse within a few months in 1989? And why did the Soviet Union itself come apart less than two years later? ... Was there some global dynamic at work that transcends individual personalities?*" Many

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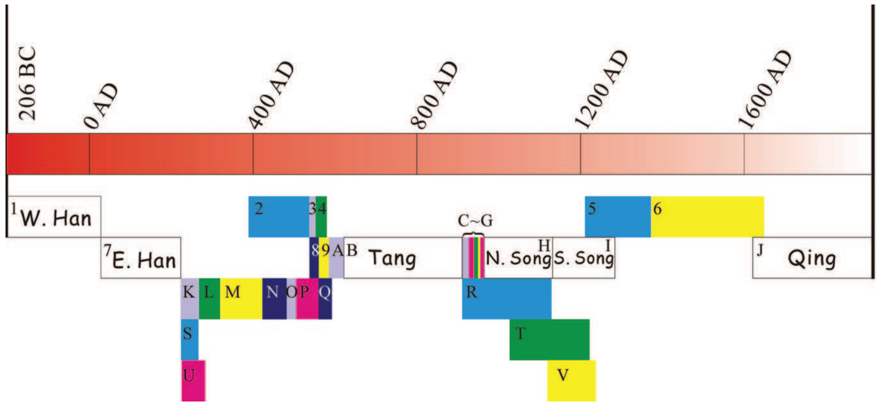

Fig. 1: (Colour on-line) A simplified chronological table for ancient Chinese history, showing clearly Han Dynasty, Tang Dynasty, Song Dynasty and Qing Dynasty. Duration data for each dynasties are (1) Western Han: 231 years; (2) Northern Wei: 148 years; (3) Eastern Wei: 16 years; (4) Northern Qi: 27 years; (5) Yuan: 107 years; (6) Ming: 276 years; (7) Eastern Han: 195 years; (8) Western Wei: 22 years; (9) Northern Zhou: 24 years; (A) Sui: 37 years; (B) Tang: 289 years; (C) Later Liang: 16 years; (D) Later Tang: 13 years; (E) Later Jin: 10 years; (F) Later Han: 3 years; (G) Later Zhou: 8 years; (H) Northern Song: 167 years; (I) Southern Song: 152 years; (J) Qing: 295 years; (K) Wei: 45 years; (L) Western Jin: 51 years; (M) Eastern Jin: 103 years; (N) Song (Northern Dynasties): 59 years; (O) Qi (Northern Dynasties): 23 years; (P) Liang (Northern Dynasties): 55 years; (Q) Chen (Northern Dynasties): 32 years; (R) Liao: 209 years; (S) Shu: 42 years; (T) Western Xia: 195 years; (U) Wu: 58 years; (V) Jin: 119 years.

dynasties, including the well-known Han Dynasty, Tang Dynasty and Song Dynasty, had been established and then collapsed in the ancient Chinese history (fig. 1). Together with information on Egyptian history, the duration data of those dynasties provides a possible approach to address such an absorbing question proposed by Waldrop. Analogous to crustal earthquakes in natural fault systems, we here consider the dynasty collapses as big shocks (extreme events) in human society. The duration of a dynasty then represents the inter-event time of two successive extreme events. To the pattern of the chronological data in the history of human being, we are highly interested in the statistical distribution of inter-event times of historical dynasty collapses and its underlying process. In this paper, by means of rank-ordering statistics, we demonstrate that the duration data of those ancient dynasties in the China and Egypt chronologies could be described with good accuracy by the Weibull distribution. Recently such distribution was used to fit the probability density function of intertrade times between consecutive stock trades of thirty companies representing eight sectors of the U.S. economy over a period of 4 years [19]. Romero *et al.* [20] found that the normalized prices of both Babylon and England agricultural commodities are characterized by stretched exponential distributions (similar to Weibull), and exhibit persistent correlations of a power-law type over long periods of up to several centuries, in contrast to contemporary markets. Since the exponential distribution is a special case of the Weibull distribution we have in this paper adopted a more general Weibull distribution, that takes on a wide variety of shapes ranging from exponential to bell-shaped, to describe those historical data. Another appeal for adopting the Weibull distribution is that the Weibull distribution is widely used in the time-to-failure statistics.

Data and method. – We focus on ancient Chinese and Egyptian chronologies in this paper because both had most distinctly political structure of a unified government/authority. 221 BC is accepted to be the year when China became unified under a large empire ruled by the Emperor Zheng Ying (`http://en.wikipedia.org/wiki/History_ of_ China`). Subsequent dynasties in ancient Chinese history developed bureaucratic systems that enabled the emperor of China to control the large territory. Similarly, in the ancient Egypt, the need to manage the waters of the Nile River led to the creation of the first political organization in the world and the year for unification of Egypt could be probably placed in about 3000 BC (`http://en.wikipedia.org/wiki/History_ of_ Egypt`). The last dynasty in China was the Qing dynasty in 1911 AD while the Ptolemaic dynasty in 30 BC was the last for Egypt. We thus analyzed sequences of 31 and 28 major dynasties in ancient Chinese and Egyptian chronologies, respectively.

The rank-ordering technique provides a robust method for retrieving the nature of the underlying distribution of extreme events, even from samples of few observations [8,21]. It offers a better adaptability to analyze the tail of a distribution of extreme events and thus represents a conservative perspective on the rarely extreme events of a population [8]. Rank-ordering statistics is initially introduced in linguistics [22] and then widely used in statistics [18]. Using the rank-ordering statistics, Bouchaud *et al.* [23] studied the Levy-like tails of return distributions of financial time series and their associated

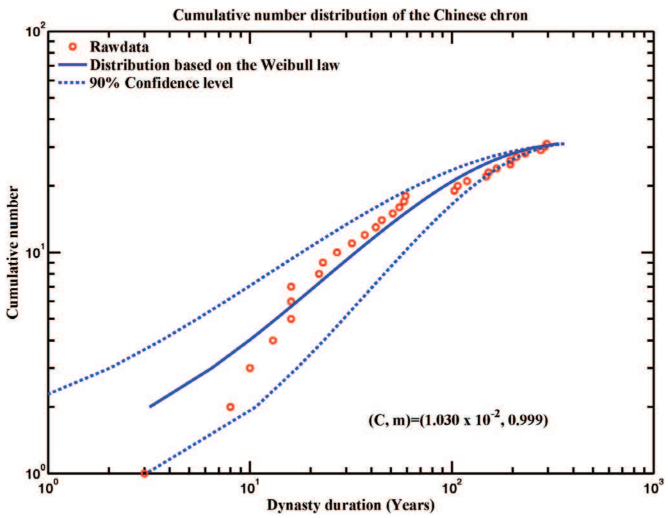

Fig. 2: (Colour on-line) Rank-ordering analysis for Chinese chronology.

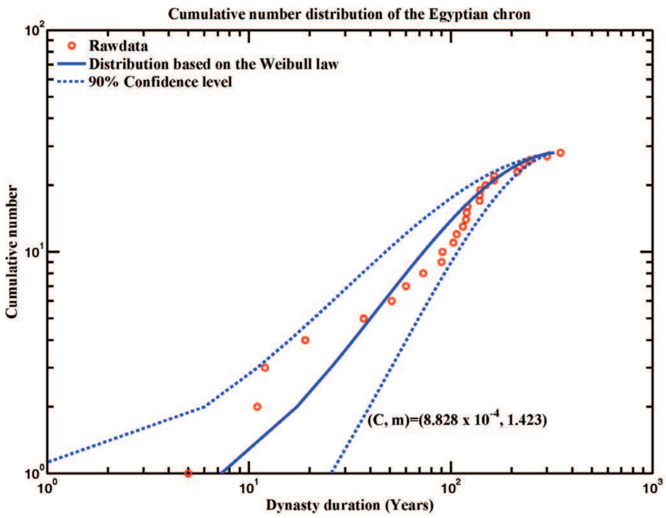

Fig. 3: (Colour on-line) Rank-ordering analysis for Egyptian chronology.

large deviations. Mantegna *et al.* [24] had analyzed the coding and noncoding regions of DNA sequences by the rank-order analysis. Also, Luongo and Mazzarella [25] applied the rank-ordering statistics to investigate the time-scale invariance of the eruptive activity of Mt. Vesuvius. We refer readers to those papers [8, 18, 21–25], together with references therein, for the generality and usefulness of rank-ordering statistics. Recently, the Zipf rank approach was used to predict the *per capita* gross domestic product in 30 years of developing and developed EU countries [26], and to reveal power laws in bankruptcy data [27] and financial accounting ratios [28].

The rank-ordering statistic is defined by ranking the duration data T_i in decreasing order ($T_1 \geqslant T_2 \geqslant \ldots \geqslant T_n$) and then analyzing T_i against rank i. The probability density function (PDF) of the ith rank, denoted by $\Phi_{i,n}(x)$, is

$$\Phi_{i,n}(x) = C_{i,n} (F(x))^{n-i} (1-F(x))^{i-1} f(x), \quad (1)$$

where $C_{i,n} = (n-i+1)\binom{n}{i}$. $F(x)$ and $f(x)$ are the distribution function and probability density function of the random variable x of interest (for instance, the duration T_i in our question). Let us consider $f(x)$ as the Weibull distribution with two parameters C and m,

$$f(x) = Cmx^{m-1}e^{-Cx^m}. \quad (2)$$

Then the mode $M_{i,n}$ of the PDF $\Phi_{i,n}(x)$ is the most probable value of the random variable x. By substituting (2) into (1) and differentiating $\Phi_{i,n}(x)$ with respect to x, an implicit relationship for $M_{i,n}$ can be obtained as the following:

$$(n-i) - (i-1) F(M_{i,n}) / (1 - F(M_{i,n})) - (m-1)/ \\ CM_{i,n} + mM_{i,n}^{m-1} = 0. \quad (3)$$

Provided some prescribed confidence level, the confidence interval around the mode can be also derived from $\Phi_{i,n}(x)$. Given the Weibull distribution with, say, the maximum likelihood estimates of two parameters C and m, we thus calculate the confidence intervals, *i.e.* ($C_\pm$, $m_\pm$), for the two Weibull parameters. Then we can obtain two Weibull distribution curves with these two sets of parameters, *i.e.* (C_+, m_+) and (C_-, m_-). The two curves represent confidence interval of cumulative number for specific dynasty duration T_i.

Results. – The results of the rank-ordering analysis applied to the dynasty duration data of the Chinese chronology are shown in fig. 2 and those for the Egyptian one in fig. 3. Each plot represents the cumulative number of dynasties as a function of duration T in years, *i.e.* the number of dynasties with the duration equal to or smaller than T.

The Weibull parameters (C, m) were estimated by means of the maximum likelihood method [29]. For the Chinese dynasty series we obtained (1.030×10^{-2}, 0.999), while for the Egyptian one we obtained (8.828×10^{-4}, 1.423). The plots show also that both the distributions fall within the 90% confidence intervals, indicating that the Weibull distribution is a good model for the dynasty series. In order to verify whether the results are not influenced by possible errors in the determination of the duration of a dynasty, we added 10% random noise to the dynasty duration data and calculated the Weibull parameters. The mean values of the shape parameter m for Chinese and Egyptian datasets over 500 random realizations are 0.998 and 1.419, respectively, with standard deviations of 0.011 and 0.026, respectively. Therefore, the results are not influenced by possible bias in the determination of the duration of the dynasty.

In order to increase the sample size and estimate the Weibull parameters more accurately, we merged the

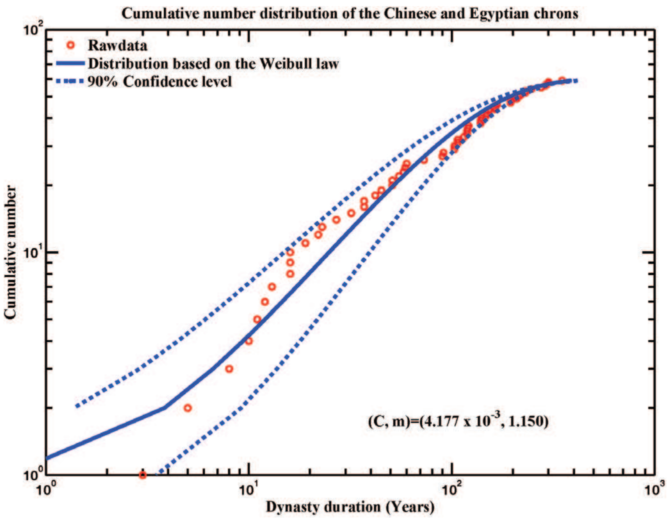

Fig. 4: (Colour on-line) Rank-ordering analysis for merged Chinese and Egyptian chronologies.

Chinese and Egyptian datasets. But before merging them, we verified their statistical similarity using methods, which are independent of rank-ordering statistics. Furthermore, both raw datasets without ranking are examined. We applied the two-sample t-test to verify whether the means are significantly equal. The obtained p-value is 0.19, indicating the two means are equal with a confidence level of at least 95%. The two-sample F-test could be also applied to compare the variances of the two datasets. The obtained p-value of 0.74 indicates that the variances of the two datasets are equal with a confidence level of at least 95%. Both hypothesis tests indicate that the Chinese and Egyptian datasets are highly likely resulted from the same population distributed with the same mean and variance. Therefore, we can conclude that the Chinese and Egyptian datasets are drawn from the same population. We can furthermore consider the one-way analysis of variance (ANOVA) to study if those datasets of China, Egypt and the hybrid one are from the same population. Because these three sets have different sample sizes and the minimum one is 28, we randomly select 25 samples from each of them to relieve sample size effects for analysis. A thousand of trials are then considered and the calculated p-value is 0.35 ± 0.28, indicating that the null hypothesis that three datasets are from the same population cannot be rejected. Note that the population used in the above tests is a normal distribution. Since the hypothesis tests and rank-ordering statistics are used to analyze different forms of datasets (ranked or not), one can still expect to reach the conclusion that two sets of ranked data are collected from same population but different from the normal distribution. Therefore, it is reasonably that we can group Chinese and Egyptian datasets to increase the sample size for our study. Figure 4 shows the results of the Weibull fitting for the merged (hybrid) dataset with $(C,\ m) = (4.177 \times 10^{-3},\ 1.150)$ along with a 90% confidence band.

In order to check if the Chinese, Egyptian and hybrid dynasty chronologies are significantly different from those randomly generated based on a normal distribution, we applied the ANOVA test again and we generated one thousand of Gaussian random realizations; the p-values of the three tests are 0.06 ± 0.12, 0.00 ± 0.00 and 0.01 ± 0.05, respectively, which indicate that the three datasets are not randomly generated with a confidence level of at least 90%.

We can also calculate R-squared values of the prediction from the fitted model against three datasets to quantitatively assess the extent that the variability of the dataset to be explained by the Weibull distribution. For the real duration data of Chinese (fig. 2), Egyptian (fig. 3) and hybrid chronologies (fig. 4), the R-square values are 0.97, 0.95 and 0.98, respectively. These R-square values suggest that the variability of those three datasets can be well explained by the Weibull distribution.

Long-range connective sandpile model for the dynasty collapse. – It is definitely not easy to model the disorder of human society at this moment in time. Interestingly, though, we would here like to propose a very intuitive prototype to be a working model for the dynasty collapse. The sandpile model has been the paradigm of self-organized complex systems, even including the evolution of biological life [30]. The sandpile model can be used for modeling society. Each individual group of society can suffer a certain social/emotional stress, which is described by adding a certain amount of sand grains to each cell of the sandpile. Once the total amount of the accumulated sand grains within a single cell reaches the threshold, they will be redistributed to four nearest-neighboring cells and could thus induce a chain of topplings, which is called an avalanche event. In such model, the social event can be assimilated to the avalanche event in the sandpile and the dynasty collapse with the system-wide avalanche event. However, different from the original Bak-Tang-Wiesenfeld–type (BTW) sandpile model [30], we here utilize a long-range transfer of redistributing grains [31–34] in order to model the long-range transfer of stress among the separated social groups due to information/message delivery. Such kind of transfer is not limited to the nearest-neighboring transfer of stress (disorder) in the BTW sandpile model. The so-called *long-range connective sandpile (*LRCS) model [31–34] can thus take into account such long-range transfer of stress/disorder. Consider that the unfavorable news, say disorder somewhere in a human society, could cause upsets in people's mind at faraway places. Emotional stress is thus of *long-range influence* and need not be the nearest-neighboring one.

For a square lattice of L-by-L cells, we randomly throw sands, one at a time, onto the grid. In the original BTW sandpile model [30], once the total amount of the accumulated sands on a single cell reaches the threshold amount of four, they will be redistributed to the four adjacent cells (the nearest neighbors) or lost off the edge of the grid. The LRCS model [31–34] differs from

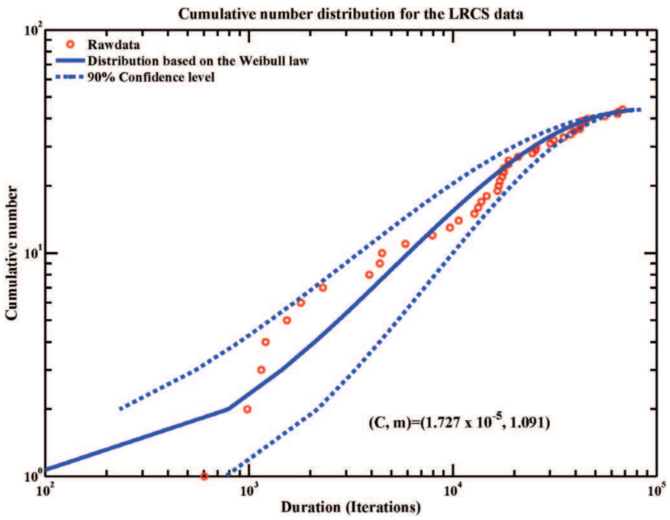

Fig. 5: (Colour on-line) Rank-ordering analysis for the inter-event time distribution of system-wide events in the LRCS model.

the BTW model in view of releasing toppled grains to four nearest-neighboring cells. The modified rule of randomly internal connections is very similar to the implementation of Watts and Strogatz [35]. For any particular cell, when the accumulated grains exceed the threshold and redistribution occurs, one of the original nearest-neighbor connections confronts a chance with the *long-range connective probability* P_c of redirecting to a randomly chosen, distant cell and so the original connection is replaced by a randomly chosen mesh that may be far from the toppling cell. We can assume that the degree of long-range influence depends in an adapted way on the size of last disorder event [31–34], which means a large perturbative event can cause a high degree of long-range influence. By using such self-adapted probability threshold P_c of remote connection, the self-adapted LRCS model demonstrates a state of *intermittent criticality*, in which the system quasi-periodically approaches and retreats from the critical state. A system-wide disorder that analogizes the dynasty collapse can frequently be observed in the LRCS model.

Figure 5 shows the results of the Weibull fitting to the LRCS-based synthetic dynasty collapse events: the inter-event time distribution obtained from 45 system-wide events can be fitted very well by the Weibull distribution with parameters $(C,\ m) = (1.727 \times 10^{-5},\ 1.091)$ together with the R-square value of 0.97. Note that the inter-event time distribution of extreme events is not Weibullian in the BTW sandpile model.

Discussion and conclusions. – The rank-ordering technique is well known in the statistical community to be useful for extracting the tail of the distribution of a sparse dataset, often characterized by under-sampled fat tails that correspond to rare extreme events. In this study, by means of applying the rank-ordering statistics, we show that historical dynasty duration can be fitted very well by a Weibull distribution. The Weibull distribution was initially proposed to describe the life length of materials under fatigue and fracture loads [36]. The Weibull theory uses the weakest link approach to describe the strength of various materials and has been successfully used in characterizing the variations of the time to failure of mechanical and electrical components and brittle materials. It is thus amazing that the distribution of the time to failure of human society follows the same process as material fails. Note that the Weibull distribution is the only distribution that gives a scale-invariant power-law hazard function [9]. The self-similar dependence of the hazard rate on time could lead directly to the Weibull distribution of failure times.

Khmaladze *et al.* [15] found that data on the lengths of rule of Roman emperors showed can be fitted by the exponential distribution. Khmaladze *et al.* thus claimed that their reigns ceased purely at random, in unexpected and unpredictable way, and not as a consequence of accumulated tensions [15]. While the durations of the reigns can be exponentially distributed following a Poissonian process without memory effect, our present analysis suggests that the Weibull distribution with long-term memory represents a reasonable fit to the durations of the dynasties. The challenge of our present analysis is then to understand how the exponential Poisson distribution of individuals is renormalized into the Weibull distribution of aggregates. What mechanism(s) could be at the origin of the observed Weibull distribution of historical dynasty duration? Is it possible that the life of a dynasty could be explained by the aging process with memory? We are not so much surprised by the applicability of such mechanism to natural earthquake fault systems. By means of the spring-block models, Abaimov *et al.* have demonstrated that the distribution of the interval times between smaller earthquakes follows an exponential Poissonian model while the distribution of recurrence times between extremely system-wide earthquakes is often Weibullian [9]. Therefore, if the fall of an emperor can be an analogue of a small event and the dynasty collapse a system-wide event, the abovementioned similarity between self-organized complex system (the slider-block system) and human society is again surprising.

During the last decade Sociophysics has been the source of many efforts for physicists to analyze the information content of social phenomena with respect to complex systems, using tools borrowed from statistics, statistical physics and nonlinear dynamics. Complex systems are characterized by a great amount of independent agents interacting with each other in a great many ways. The science of complexity including statistics, statistical physics and nonlinear dynamics, is expected as an enlightening tool to probe the behaviors of complex systems like human society. This study represents an attempt on statistical investigation of human society and suggests that there may exist a simple rule, *i.e.* the Weibull aging process, for collective human activities in history.

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