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Negative correlation between frequency-magnitude power-law exponent and Hurst coefficient in the Long-Range Connective Sandpile model for earthquakes and for real seismicity

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Abstract – The Long-Range Connective Sandpile (LRCS) model was applied to the Italian seismicity. The Hurst exponent and the power-law slope of the frequency-size distributions for the avalanches in the LRCS model and for earthquakes in Italy are investigated. This study shows the transition of the correlation coefficient between b and H values with different calculation window length. The result shows similar behaviors in the LRCS model and the Italy catalogue. The negative correlation between b and H values can be clearly seen when an appropriate window length is employed for various time series. We suggest that the negative relationship is caused by the increasing correlation length as the system accumulates enough energy. Also the calculation window length is an important index to display the intensity of the negative correlation between these two exponents. The appropriate window length can be related to the period time for the avalanches with various sandpile and seismicity time series.

Introduction. – The universal Gutenberg-Richter power-law relation, proposed in 1956 [1], states that the number of earthquakes with magnitude larger than M follows the power law $\log_{10}N_M = a - bM$, with scaling exponent b. Many studies proposed that b could be a monitoring index informing about forthcoming large events [2–10]. Also many studies reported the decrease of b before the occurrence of large earthquakes [11–14]. On the basis of laboratory experiments, it was found that the b value of foreshocks is lower than background seismicity [15].

The Hurst exponent is calculated by the well-known rescaled range statistical analysis (R/S analysis), firstly developed by Hurst [16], and commonly used to identify correlations in time series. Indicating by τ the time span, ξ_t the time series, R the difference between the maximum and minimum amounts of accumulated departure of the time series from the mean over τ, and S the standard deviation calculated over τ, the ratio R/S is called rescaled range, which for a correlated series is a power law: $R/S = (\tau/2)^H$, where H is the Hurst exponent. For $H > 0.5$ the time series is persistent (if the series increased/decreased in the past, it is very likely that it will keep on increasing/decreasing in the future). On the contrary, for $H < 0.5$ the time series is antipersistent. If $H = 0.5$, the time series is a purely random uncorrelated process.

Negative correlation between b and H in observational earthquake time series was already observed [15,17–19], but in the present study we calculate these two parameters for the avalanche series in the Long-Range Connective Sandpile (LRCS) model and for the real seismicity occurred in Italy, confirming the negative correlation between them in both cases (hereafter we will use b for real earthquake data, and B for the LRCS model).

Long-Range Connective Sandpile model. – The LRCS model is derived from the Bak-Tang-Wiesenfeld (BTW) sandpile model and represents a reasonable model for the long-range effects in real earthquake fault systems [20–23]. The BTW model, proposed by [24,25], uses a square grid of $L \times L$ cells, and sand is thrown randomly one grain at a time onto the grid. When the total number of accumulated sand grains on any single cell reaches a threshold value, the sand grains are redistributed

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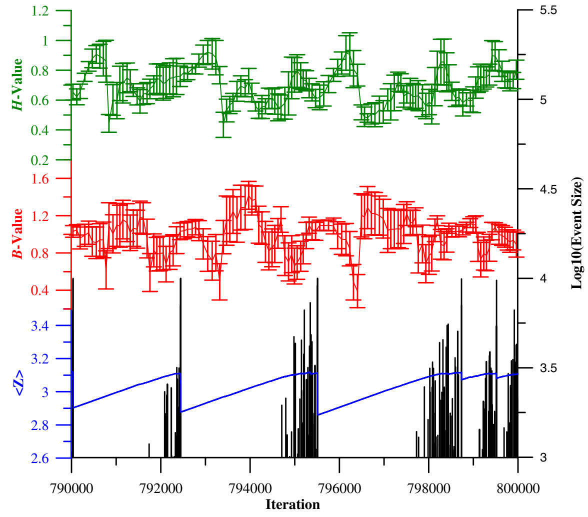

Fig. 1: (Colour on-line) LRCS model simulation for a square lattice of 100×100 cells with time window length equal to 300. The blue line represents the dynamic variable *Z(t)* of the average topographic height of the LRCS model. The green and red lines are the Hurst exponent H of avalanche sizes and the power-law exponent B of the frequency-size distribution, respectively. Error bars show the 95% confidence intervals. Also shown are the time occurrences of avalanches with sizes > 3162 (black bars) [23].

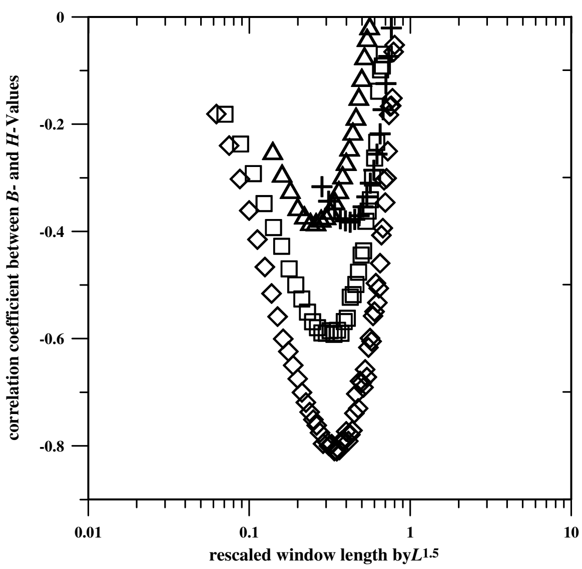

Fig. 2: Correlation coefficients between B and H values calculated with different window lengths for LRCS models. The system sizes L range from 50 to 400 (crosses for $L = 50$; triangles for $L = 100$; squares for $L = 200$; diamonds for $L = 400$). The window lengths for the different grid sizes of LRCS models are rescaled by $L^{1.5}$.

onto the four adjacent cells (the nearest neighbours). In the case in which redistributed adjacent cells are off the edge of the grid, the sand grains are lost.

The LRCS model is similar to the BTW model, but when releasing sand grains upon reaching the threshold value, the grains are redistributed according to random connections within the whole $L \times L$ grid. The modified rule of random internal connections for redistribution is similar to the model proposed by Watts and Strogatz [26]. When the accumulated number of grains exceeds the threshold for a cell, redistribution will occur: one of the original four grains has a chance of being replaced with long-range connective probability Pc by a randomly chosen cell, which may be far from the redistributing cell. The redistribution of sand grains in the LRCS model can be considered, then, as a way of releasing energy with long-range effects like in real earthquake fault systems. The Pc depends strongly on the topographic change induced by the latest event [20–23], which had simply been defined as $Pc\ (i+1) = [\Delta Z(i)/\alpha L^2]^3$ ($\alpha \sim 1.25$). $\Delta Z(i)$ and L^2 are the topographic change due to the latest event and the system size, respectively (for the details of the LRCS model, the reader can refer to [20–23]). The LRCS model displays the same characteristics of power-law frequency-size distribution.

Using the LRCS model, we calculated the time variation of the power-law exponent B of the frequency-size distribution and of the Hurst exponent H of the event sizes. We used a window of a fixed number of events (indicated as window length hereafter) sliding through the entire series with a shift of 10% of the window length. The window length varied from 100 to 4000 events. Figure 1 shows an example of a 100×100 cell grid [23].

Figure 2 shows the correlation coefficients between B and H for different window lengths in the LRCS model with different grid size in order to take into account the finite-size effects: 50×50, 100×100, 200×200 and 400×400. Figure 2 shows the results after rescaling the horizontal axis (the window length) $L^{1.5}$, where L indicates the grid size. It is visible that B and H are maximally negatively correlated for a rescaled window length of $\sim$0.3. The correlation coefficient ranges between -0.4 and -0.8 for L, increasing from 50 to 400. Since the same behaviour is observed independently of the system sizes L, the negative correlation cannot be due to finite-size effects [27], and can be considered a robust feature of the LRCS model.

Analysis of the Italian seismicity. – Is negative correlation between b and H also present in real earthquake data? To answer this question, we analyzed the Italian earthquake catalogue. Figure 3 shows the Italian seismicity studied in the present paper. The catalogue spans from 2005 to 2010. The frequency-magnitude distribution is shown in fig. 4. The earthquakes with magnitudes greater than 1 can fit the Gutenberg-Richter law. The b-value of the Gutenberg-Richter law is $\sim$1.0, and the completeness magnitude can be considered equal to 1. To avoid bias by aftershocks, the catalogue was firstly de-clustered by using the double-link method of Wu and Chiao (2006) [8]. We applied the data binning technique proposed by Christensen and Moloney [28] to reduce the noise effect of large avalanches (*i.e.*, the effect of finite

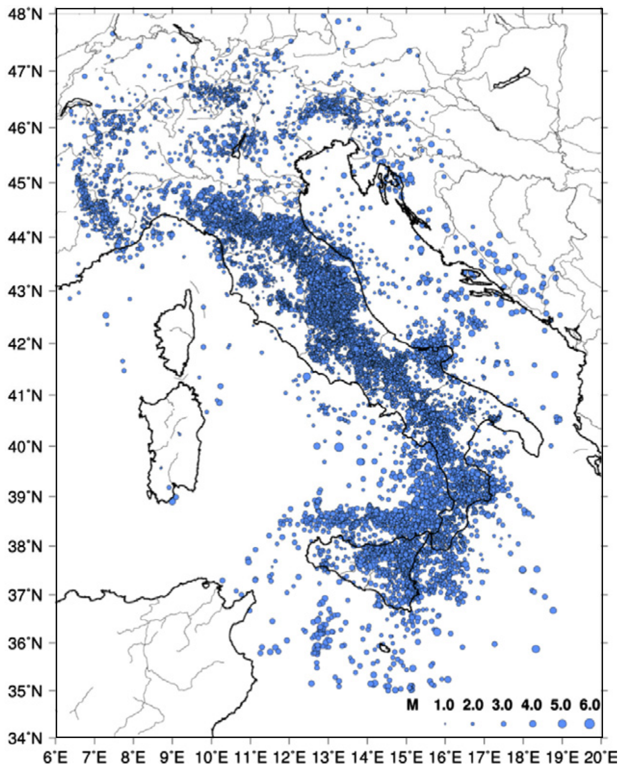

Fig. 3: (Colour on-line) Epicenters of earthquakes in the area of Italy included in this study. The map shows earthquakes with magnitudes greater than 1.

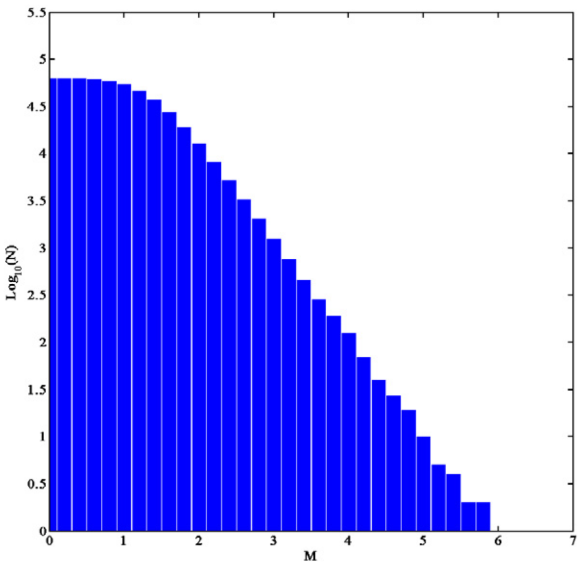

Fig. 4: (Colour on-line) Frequency-magnitude distribution of earthquakes in the Italy catalogue.

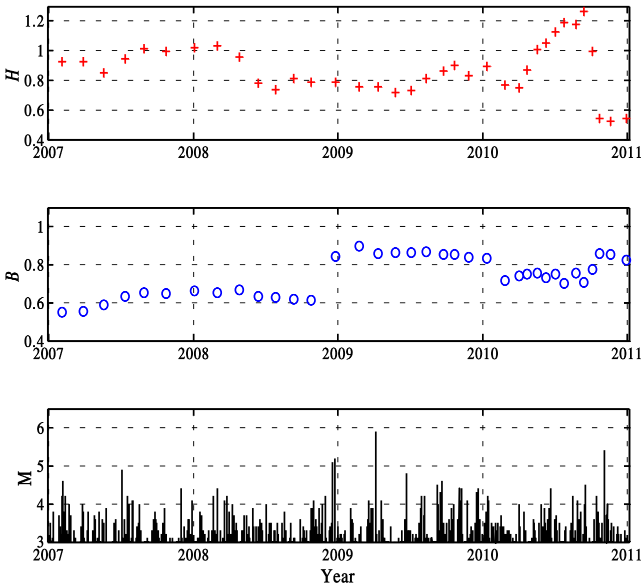

Fig. 5: (Colour on-line) Temporal variation of b (blue circles) and H (red crosses) values, calculated with 8200 events and shift of 820 events in the time span from 2007 to 2011 of the Italian seismicity.

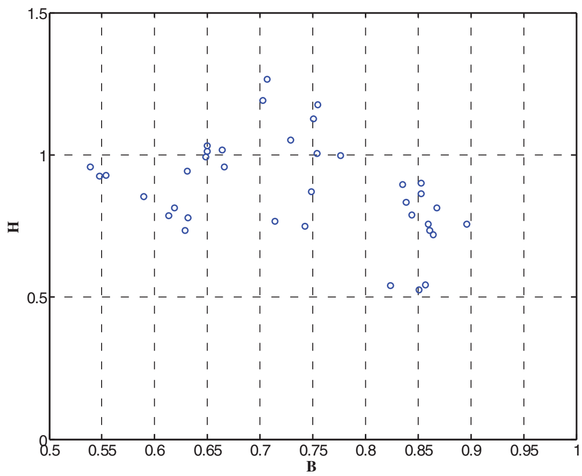

Fig. 6: (Colour on-line) Relationship between b and H, as plotted in fig. 5.

statistics). We then performed a least-squares regression to fit the frequency-size distribution. The temporal variation of b and H values for a window of ∼8200 events is shown in fig. 5 and their relationship in fig. 6. The relationship between b and H for the Italian catalogue is shown in fig. 7. The blue circles indicate the correlation coefficient between b and H varying the rescaled window length from 3000 to 10000 earthquakes. Such relationship is essentially identical to that obtained in the LRCS model. For a window length of ∼8200, the relationship between b and H shows the maximally negative correlation of about −0.38.

From the results shown in fig. 7 we can infer that for short window lengths the relevant events may be divided into different temporal windows and the correlation between B (b) and H may be weak with small absolute values for the correlation coefficient. While for

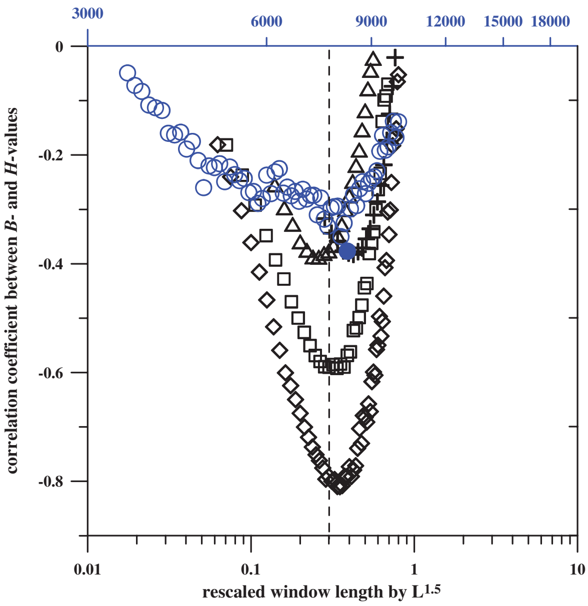

Fig. 7: (Colour on-line) The block symbols are the same as in fig. 2. The blue circles show the correlation coefficients between B and H values calculated with different window lengths (upper horizontal axis) for real earthquake data registered in the Italy catalogue. The solid blue circle shows the lowest value of the correlation coefficients between B and H values calculated from the Italy catalogue.

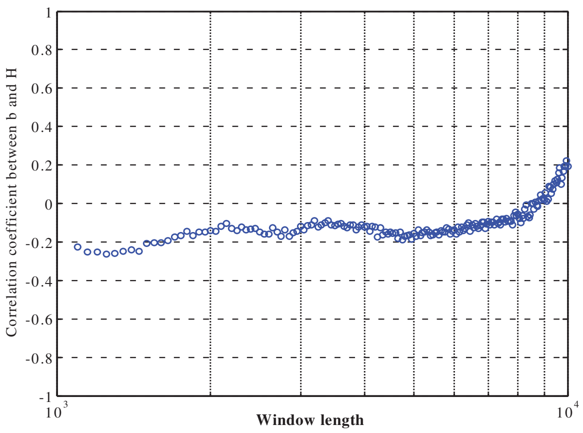

Fig. 8: (Colour on-line) Correlation coefficients between B and H values for the whole Italian catalogue.

long window lengths irrelevant events could be included in the same window, and the absolute values of the correlation coefficient may be small as well. But, for a proper window length, the negative correlation between B (b) and H can be observed; such proper window length is dependent on the size of the studied system or studied area. The larger system (the size $L \times L$) is relative to larger window length (fig. 2). We suggest that the appropriate window length can be related to the period time for the avalanches with various sandpile and seismicity time series.

We further checked our results on the whole catalogue (without removing the aftershocks) (fig. 8) and on

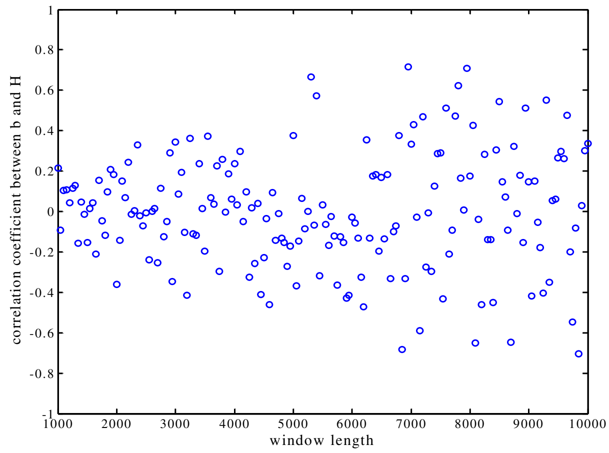

Fig. 9: (Colour on-line) Correlation coefficients between B and H values for the shuffled Italian catalogue.

a shuffled one (fig. 9). In the first case we found a correlation coefficient approximately independent of the window length. In the second case we obtained a quite disordered and scattered relationship between the correlation coefficient and the window length. These last further results strengthen our findings.

Conclusions. – In the present study we found that a real earthquake dataset is characterized by a similar negative correlation between the Hurst exponent and the power-law exponent of the frequency-size distribution as for the LRCS model. In both the model (with different system size L) and real seismic dataset it was found an almost identical rescaled window length corresponding to the maximal absolute value of the correlation coefficient. As the system accumulates energy, the average cluster size of avalanches as a cell reaches a threshold value will be getting larger and larger. The larger the average cluster size of avalanches, the larger the chance to induce medium or large events, which in turn have more chance to long-range redistribute their energy and induce more large events. This implies a lower b value. Due to such long-range triggering, the system is more persistent and shows a higher H value of the event time series.

The results of the present study also suggest that the appropriate window length can be related to the period time of the event time series of various sandpile and earthquake event time series in the Italy area.

Our findings can suggest that the LRSC model is well suited to model the real seismicity occurrence.

* * *

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