Journal of Asian Earth Sciences 77 (2013) 77–82

Contents lists available at ScienceDirect

Journal of Asian Earth Sciences

journal homepage: www.elsevier.com/locate/jseaes

Informational analysis of apparent Earth's resistivity time series to assess the reliability of magnetotelluric measurements

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ARTICLE INFO

Article history:
Received 19 April 2013
Received in revised form 26 July 2013
Accepted 7 August 2013
Available online 26 August 2013

Keywords:
Magnetotellurics
Median absolute deviation
Fisher Information Measure
Shannon entropy

ABSTRACT

We used the informational methods of the Shannon entropy power and the Fisher Information Measure to analyze the time dynamics of one year of semi-daily values of apparent resistivity, calculated on the base of the magnetotelluric (MT) monitoring at Penghu Island, Taiwan. The performed statistics indicate that the day-time measurements present higher disorder and lower organization than the night-time measurements. Such difference is more evident for the high-frequency band of resistivity data, which suggests that possibly some source of the electromagnetic field could be of anthropic origin. Furthermore, we also note the characteristics of so-called MT dead-band with the periodicity of ~10 s could be revealed in the high values of Shannon entropy power.

1. Introduction

The magnetotelluric (MT) method is a non-invasive technique measuring the variation of the electromagnetic (EM) field on the Earth's surface for mapping the subsurface structures (Chen and Chen, 1998; Bahr and Simpson, 2005; Bertrand et al., 2012). The measured component of the electric and magnetic field can be related to the distribution of the electrical properties into the subsoil by:

$$\begin{bmatrix} E_x(\omega) \\ E_y(\omega) \end{bmatrix} = \begin{bmatrix} Z_{xx}(\omega) & Z_{xy}(\omega) \\ Z_{yx}(\omega) & Z_{yy}(\omega) \end{bmatrix} \begin{bmatrix} H_x(\omega) \\ H_y(\omega) \end{bmatrix}, \tag{1}$$

in which E and H are co-located electric and magnetic fields and Z is the MT impedance tensor. Conventionally, the x direction could be referred to the north. The above formula holds assuming that the source fields are plane waves of infinite horizontal extent; such assumption can be considered reasonable if the EM skin depth is small compared to source length scales (Egbert et al., 2000). The off-diagonal elements of Z are used to obtain the apparent resistivity estimates useful to study the in-depth distribution of the electrical properties of the subsoil:

$$\rho^a_{ij}(\omega) = \frac{1}{\mu_0 \omega} |Z_{ij}(\omega)|^2. \tag{2}$$

In recent years, the MT method proved to be effective in a broad range of applications regarding the Earth's interior studies. Besides the well-known applications, i.e., structure, geometry and physical state of the crust and upper mantle (see Bahr and Simpson, 2005; Bedrosian, 2007 and references herein), continuous MT monitoring aiming at revealing seismo-related changes in the Earth's conductive structure represents one of the most challenging issues (Eisel and Egbert, 2001; Kappler et al., 2010; IPOC project, http://www.i-poc-network.org/). Furthermore, MT monitoring is used for geotectonic pattern and recognition of possible seismoelectric signal (Hadjioannou et al., 1993; Vallianatos and Eftaxias, 1993; Vallianatos, 1996, 2002; Nomikos and Vallianatos, 1997; Tzanis et al., 2000), which represented an important task in the field of earthquake precursory signals (Colangelo et al., 2000; Telesca et al., 2004).

Apart from seismo-related effects, several dynamics have been demonstrated to influence the temporal stability of an MT monitoring. Significant seasonal components of variability were observed in the Parkfield MT monitoring (Kappler et al., 2010) with a tendency for the apparent resistivity estimates to be less stable during the rainy season, as well as, to step upwards with the onset of the rain. Balasco et al. (2007, 2010) showed the possible link between the MT monitoring results and the seasonal modifications of the electrical properties in the shallowest strata of the subsoil, due to periodical cycle of drying and wetting of the Agri Valley (Italy) water reservoir. Other instability factors are the electromagnetic noise in the monitored area; the source of the MT field becomes

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1367-9120/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jseaes.2013.08.010

a limiting factor in urbanized areas, where the ratio between the "natural" signal and the cultural noise is relatively low. Although processing techniques aiming at reducing the cultural noise, it is not always possible to totally suppress such effects that can produce misleading estimation of the distribution of the electrical properties within the subsoil.

Therefore, due to the different sources of variation of the electromagnetic field on the Earth's surface, it is necessary to apply methods able to deeply investigate the inner time structure of such signals, which are obviously very complex and heterogeneous. In this scientific context, the analysis of the time series of MT apparent resistivity could be useful in terms of identifying the ranges of sounding periods in which the resistivity is more or less sensitive to different factors. Furthermore, the analysis of the time variation of the apparent resistivity in different time ranges (for instance during day-time or night-time) could also contribute to a better understanding of the possible sources of variation.

In the present paper, we aim at investigating one year of daily time series of resistivity calculated through the MT method in Taiwan. In particular, we will use the informational methods of the Fisher Information Measure (FIM) and the Shannon entropy, that, although well-known statistical methods, were not extensively used in MT analysis (Fisher, 1925; Frieden, 1990). These methods allow to gain into insight the inner time structure of the experimental signals. The FIM quantifies the amount of organization or order in a system, while Shannon entropy measures the degree of uncertainty or disorder in a system. Firstly introduced by Fisher (1925) in the context of statistical estimation, the FIM was used by Frieden (1990) to describe the evolution laws of physical systems (Martin et al., 1999). Applications like, for example, the characterization of the temporal fluctuations of electroencephalograms (EEG) and the detection of significant dynamical changes (Martin et al., 2001) were performed. The FIM was used to investigate complex geophysical and environmental phenomena (Lovallo and Telesca, 2011; Telesca and Lovallo, 2011; Telesca et al., 2011) and to reveal precursory signatures of critical phenomena (Telesca et al., 2010, 2009). The Shannon entropy, instead, is used to quantify the uncertainty of the prediction of the outcome of a probabilistic event (Shannon, 1948), being zero if such prediction is exact, and, consequently for deterministic events.

In the present study, we use these two statistical approaches to assess the reliability of the MT estimates.

2. MT site and data statistics

A suitable MT remote station was installed in Penghu Island, Taiwan for TAiwan Integrated GEodynamics Research (TAIGER) project (Bertrand et al., 2009, 2012; Chiang et al., 2010). The Penghu Island is located at an area with relatively low seismic activity and basaltic columns could be found around the island. The MT site has been already used by Bertrand et al. (2009, 2012), Chiang et al. (2010) to study the deep electrical structure in Taiwan. In beginning of 2010 a permanent and continue MT monitoring station was installed by Department of Earth Sciences, National Central University, Taiwan at the same location (Fig. 1), equipped by two 100-m-long electric dipoles and three magnetic induction coils to measure two horizontal components of the electric and three components of the magnetic fields, respectively. The MT data analyzed in this paper were recorded with 15-Hz sampling rate continuously, and also 2 s of 150 Hz and 16 s of 2400 Hz data per 3 min by the MTU-Net system (Phoenix Ltd., Canada).

In our investigation, we analyzed the semi-daily time series of apparent resistivity in Penghu site from January to December 2011, calculated in both directions *xy* and *yx* using Eq. (2), corresponding to 76 sounding periods from 3.51×10^{-3} s to 1.456×10^{3} s. We analyzed 76 couples of time series of resistivity calculated during day-time (from 7:00 to 19:00) and night-time (from 19:00 to 7:00) in order to assess the stability of the MT signals in different time intervals of a measuring day.

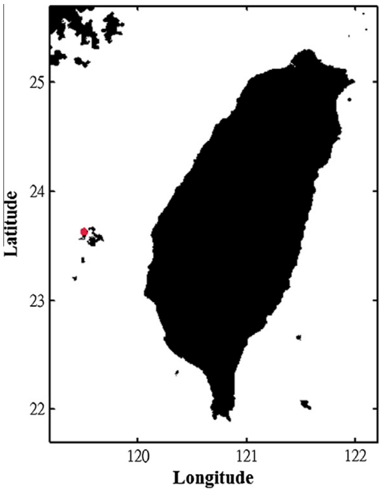

Fig. 1. Map showing the location of the MT station (red dot) in Penghu Island (Taiwan). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

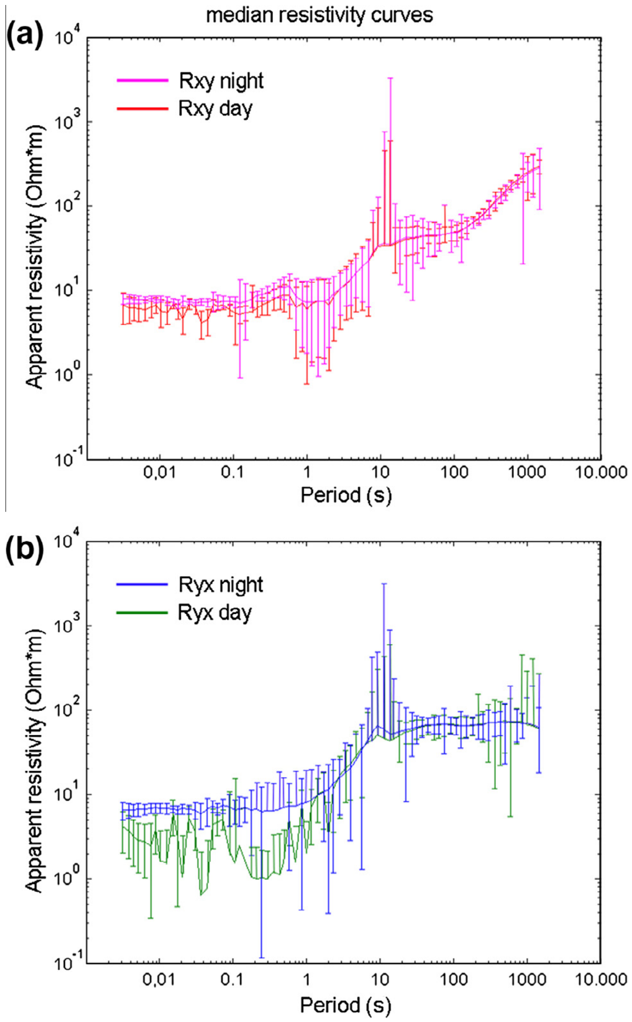

Fig. 2. Median resistivity curves calculated for the two semi-daily time series in *xy* (a) and *yx* (b) directions.

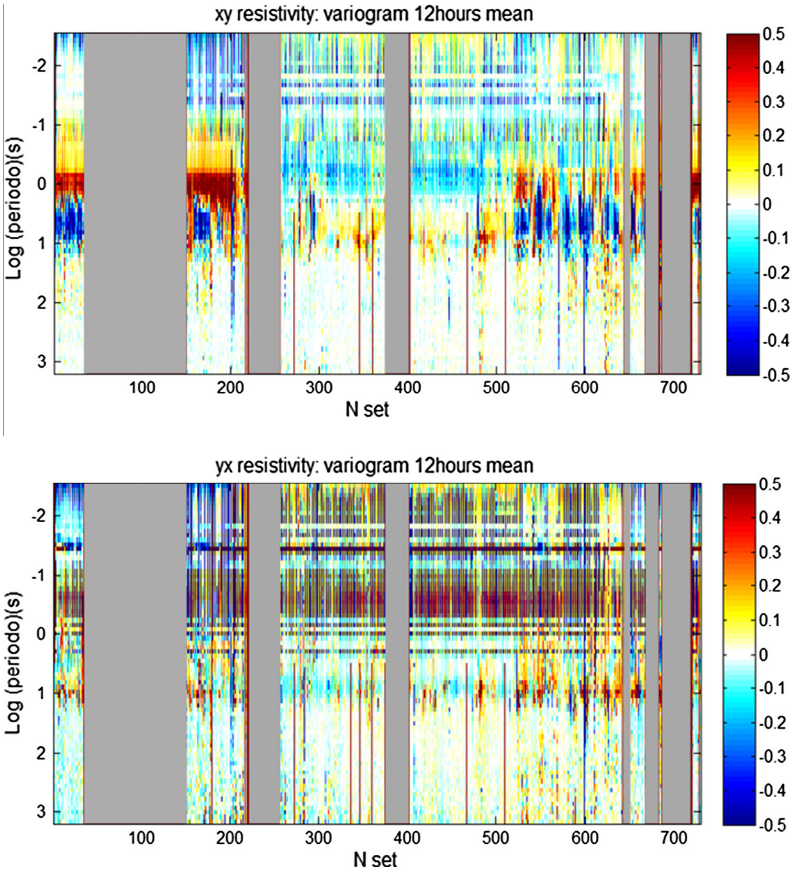

Fig. 3. Variogram of resistivity in *xy* (a) and *yx* (b) directions. The shadow regions are data missings.

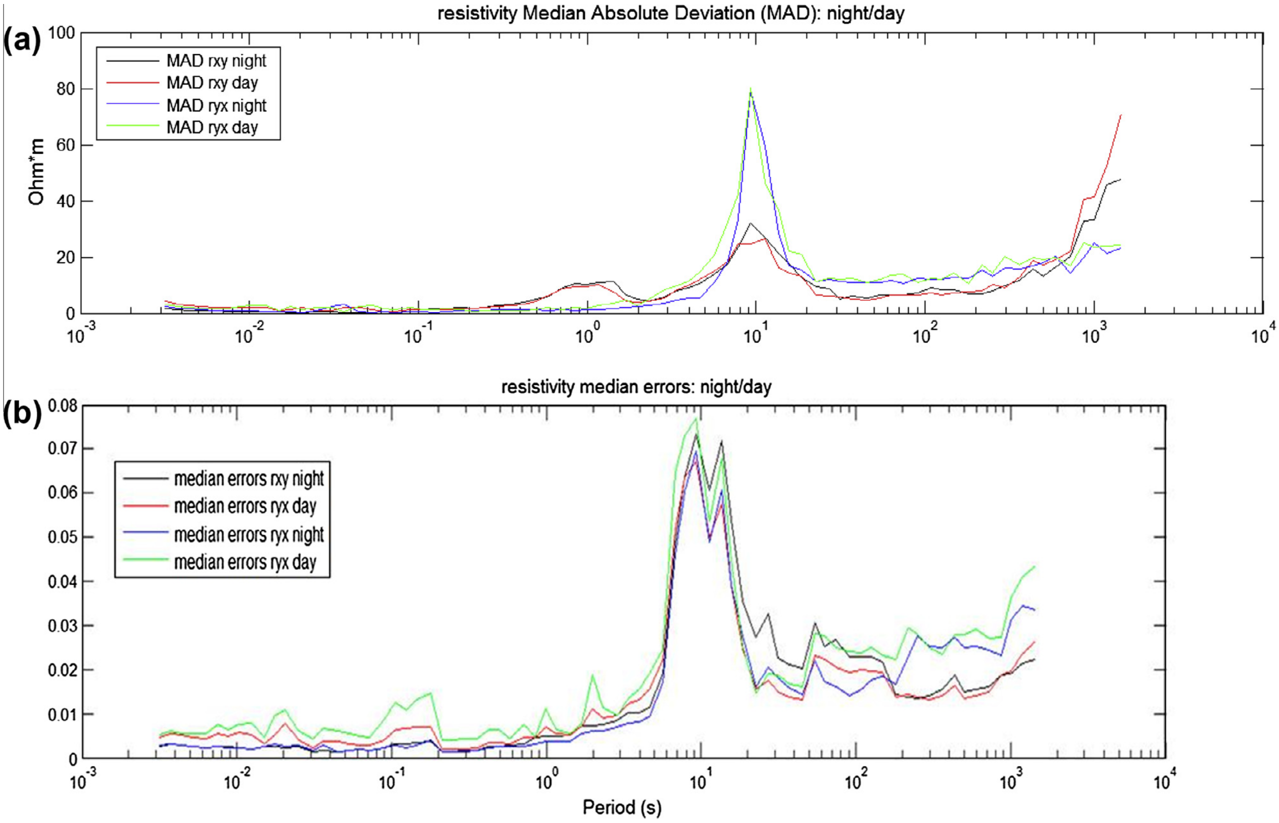

Fig. 4. (a) Median absolute deviation (MAD) and (b) median errors, for the semi-daily resistivity time series.

A preliminary analysis of the series was performed by calculating the median resistivity curve (each value of the median curve corresponding to a specific sounding period T is given by the median among all the 365 semi-daily values of resistivity at the same sounding period) for the day-time and night-time data (Fig. 2). The median curves corresponding to the day-time estimates show a quite rough shape for sounding periods T shorter than 2 s, especially in *yx* direction; furthermore in this period range the curves obtained during day-time do not coincide with those obtained during night-time. For periods T longer than 2 s, the night-time and the day-time curves seem very close.

The stability of the MT estimates was evaluated by means of the following quantity:

$$\Delta\rho_i(T) = \frac{(\log \rho_i(T) - \log \rho_c(T))}{\log \rho_c(T)} \tag{3}$$

where $\rho_i(T)$ is the semi-daily resistivity value and $\rho_c(T)$ is the long-term average computed as the median value of all the apparent resistivity estimates at each sounding period T. $\Delta\rho_i(T)$ is, then, defined as the normalized departure of the estimate from the median, and can be considered as a measure to quantify the instability of the estimate: larger the value of $\Delta\rho_i(T)$, larger the instability of the estimate.

Fig. 3 shows $\Delta\rho_i(T)$ is the *yx* component seems noisier than the *xy* component and also more unstable. The instability is higher for shorter sounding periods T ($<\sim 2$ s). Moreover a periodic structure can be observed with an increase of the resistivity values during night time. Still, for longer periods, i.e., $T > \sim 2$ s, the night-time and the day-time curves seem very close.

Fig. 4 shows the median absolute deviations (MAD) (Fig. 4a) and percentage error median (Fig. 4b) related to day-time and night-time estimates. Median errors and MAD, calculated as

$$\mathrm{MAD}(T) = median(|\rho_i(T) - \rho_c(T)|), \tag{4}$$

furnish indication respectively about the reliability of the estimates and their stability.

Percentage error median, generally larger for day-time estimates, do not show any clear increase for $T < 2$ s, indicating the possible presence of coherent noise in the MT estimates which leads to biased estimates. The peak observed around 10 s indicates a low estimate reliability, confirmed by the peak around 10 s also in the MAD plot.

3. Methods and results of FIM and Shannon entropy power

A further analysis of our resistivity time series was performed by using the FIM and the Shannon entropy power. If $f(x)$ is the probability density of a signal x, its FIM I is given by

$$I = \int_{-\infty}^{+\infty} \left(\frac{\partial}{\partial x} f(x)\right)^2 \frac{dx}{f(x)}, \tag{5}$$

and its Shannon entropy is defined as (Shannon, 1948):

$$H = -\int_{-\infty}^{+\infty} f(x) \log f(x) dx. \tag{6}$$

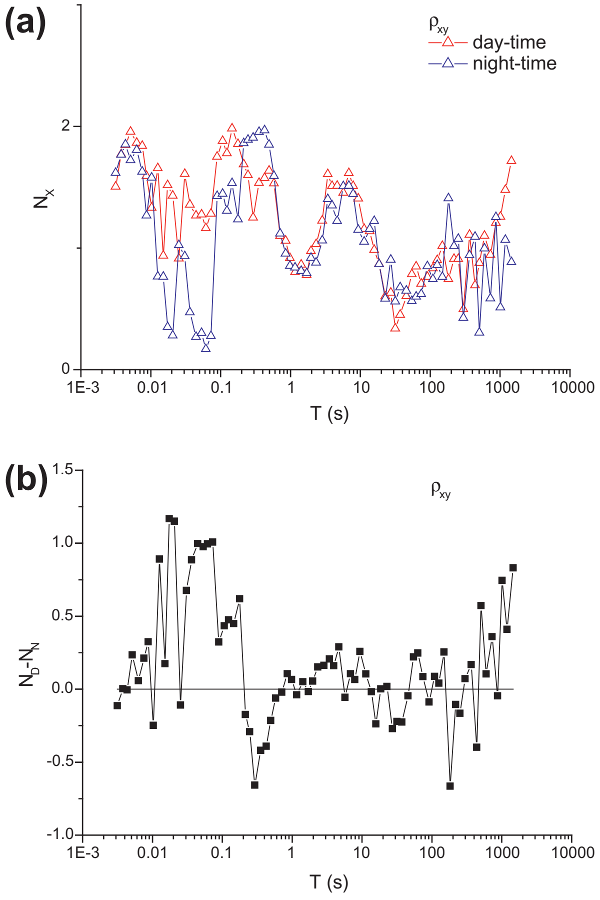

Fig. 5. Shannon entropy power for the day-time and night-time resistivity series in *xy* direction (a) and their difference (b).

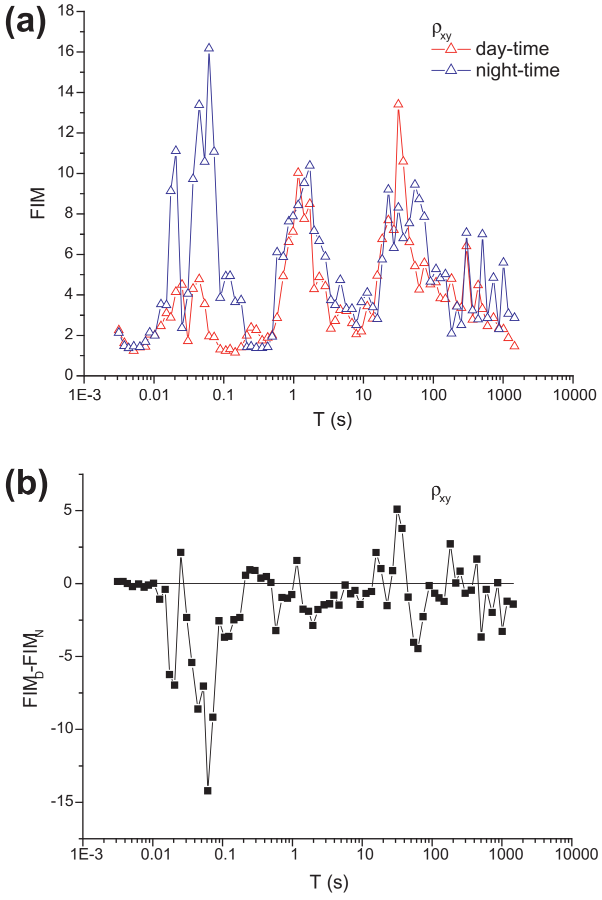

Fig. 6. FIM for the day-time and night-time resistivity series in *xy* direction (a) and their difference (b).

Alternatively the notion of Shannon entropy power can be used

$$N_X = \frac{1}{2\pi e} e^{2H}, \tag{7}$$

which satisfies the so-called 'isoperimetric inequality' $IN_X \geqslant D$ (Esquivel et al., 2010) where D is the dimension of the space. The 'isoperimetric inequality' indicates that FIM and Shannon entropy power are linked to each other. The calculation of the FIM and the Shannon entropy depends on the calculation of the probability density function $f(x)$. The pdf can be estimated by means of the kernel density estimator technique (Devroye, 1987; Janicki and Weron, 1994) that approximates the density function as

$$\hat{f}_M(x) = \frac{1}{Mb}\sum_{i=1}^{M} K\left(\frac{x - x_i}{b}\right), \tag{8}$$

with b the bandwidth, M the number of data and $K(u)$ the kernel function, a continuous non-negative and symmetric function satisfying the two following conditions

$$K(u) \geq \text{ and } \int_{-\infty}^{+\infty} K(u)du = 1. \tag{9}$$

.

In our study, we estimated the pdf $f(x)$ by means of the algorithm developed in Troudi et al. (2008) combined with that developed in Raykar and Duraiswami (2006), that uses a Gaussian kernel with zero mean and unit variance:

$$\hat{f}_M(x) = \frac{1}{M\sqrt{2\pi b^2}}\sum_{i=1}^{M} e^{-\frac{(x-x_i)^2}{2b^2}}. \tag{10}$$

Fig. 5a shows the Shannon entropy power N_X of day-time and night-time resistivity time series in *xy* direction, ρ_{xy}: the pattern of both series is very complex and characterized by large fluctuations with respect to the sounding period. Fig. 5b shows the difference between the two series plotted in Fig. 5a, namely N_D–N_N, where N_D indicates the N_X of the day-time series and N_N that of the night-time series; the difference can reveal more clearly those ranges of the sounding periods characterized by high difference in N_X between the day-time and night-time series and those where such difference is negligible. As a general observation, the Shannon entropy power for the day-time series is larger than that of the night-time series over the most of the sounding periods, and this suggests that the day-time series would be characterized by more uncertainty than the night-time series.

Fig. 6a shows the FIM of the ρ_{xy} calculated during day and night, while Fig. 6b shows the difference FIM$_D$–FIM$_N$, with the subscripts having the same meaning as above. The difference FIM$_D$–FIM$_N$ indicates a higher order degree for the night-time series for most of the sounding periods.

Also, the N_X and FIM patterns of the day-time and night-time daily time series of ρ_{yx} show, in particular, that the N_X of the day-time series is higher than that of the night-time series over almost all the period range (Fig. 7), while the FIM during day-time is generally lower than that during night-time (Fig. 8). Both the Shannon entropy power and the FIM show clear difference between day-time and night-time measurements especially for sounding periods less than 2 s; while for the higher sounding periods, such difference become less evident.

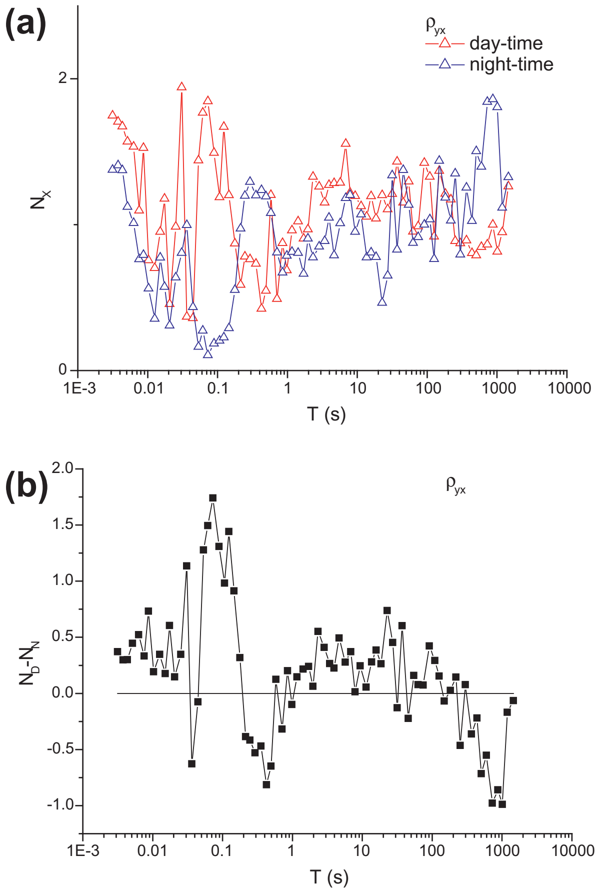

Fig. 7. Shannon entropy power for the day-time and night-time resistivity series in *yx* direction (a) and their difference (b).

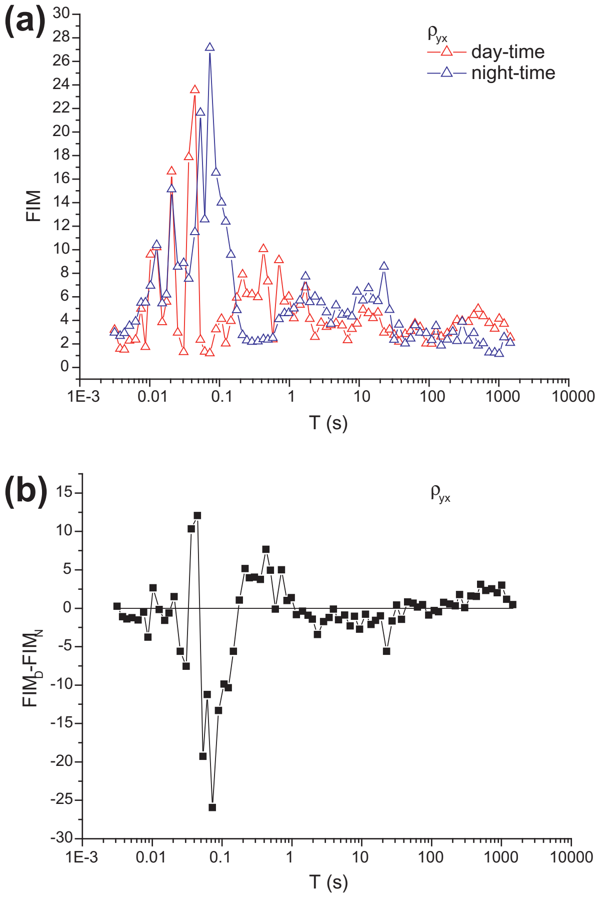

Fig. 8. FIM for the day-time and night-time resistivity series in *yx* direction (a) and their difference (b).

4. Conclusions and discussion

In the present paper we applied several statistical methods, standard and advanced, to assess the reliability of magnetotelluric measurements acquired in Penghu Island, Taiwan, during 1 year of observation. The magnetotelluric method allows us to calculate the apparent resistivity of the Earth at different sounding depths down to the seismogenic ones. Therefore, careful analysis should be performed in order to get the best estimates of resistivity. In fact, some reports (e.g., Li et al., 2009) showed that the error in MT data may lead to some in-negligible effect for MT inversion results.

In our study, we found that the daily resistivity time series are characterized by the following features:

(1) The performed statistics suggest that during day-time the measurements are characterized by higher disorder and larger amount of uncertainty (higher Shannon entropy power), while during night-time the resistivity time series are characterized by higher order (higher FIM).

(2) Although the dynamical characteristics evidenced in the previous point involve approximately all the sounding periods, it seems that the behavioral difference in terms of informational properties of the day-time and night-time series is more pronounced for the higher frequency band than the lower ones (in particular corresponding to sounding periods less than 2 s). As for the sounding periods larger than 2 s, the results of the N_X and FIM analyses look quite stable and do not show evident difference between the night-time and the day-time data.

From above, it is likely that in Penghu Island the electromagnetic field could have also an anthropic source, especially evident in the day-time measurements of high frequency data larger than 0.5 Hz. The informational statistical methods that were used in the present study have led to the identification of the time intervals of the day and the sounding periods that could be very probably influenced by noisy anthropic effects. We also note that for sounding periods larger than 2 s the Shannon entropy is higher in both the day-time and night-time data around the period of $\sim$10 s in the *xy* direction. Since this feature is common to both the semi-daily data, we suggest such high Shannon entropy could be related to the relatively high complexity of electromagnetic source fields in the so-called MT dead-band which is from about 10 s to 10 Hz where natural signals are typically weak.

Acknowledgement

The present study was supported by the Project CNR/NSC 2012-2013 “Advanced time series analysis tools for magnetotelluric data”.

References

Bahr, K., Simpson, F., 2005. Practical Magnetotellurics. Cambridge University Press, Cambridge, UK.

Balasco, M., Lapenna, V., Romano, G., Siniscalchi, A., Telesca, L., 2007. Extracting quantitative dynamics in Earth’s apparent resistivity time series by using the detrended fluctuation analysis. Physica A 374, 380–388. http://dx.doi.org/10.1016/j.physa.2006.07.028, ISSN: 0378-4371.

Balasco, M., Giocoli, A., Lapenna, V., Romano, G., Siniscalchi, A., Telesca, L., Tripaldi, S., 2010. Dynamics of internal and external origin revealed by a single-site magnetotelluric monitoring. In: SEG Expanded Abstracts, vol. 29, Denver (USA), 17–23 Ottobre 2010SEG.

Bedrosian, P., 2007. MT+, Integrating magnetotellurics to determine Earth structure, composition and processes. Surveys in Geophysics 28, 121–267. http://dx.doi.org/10.1007/s10712-007-9019-6.

Bertrand, E.A., Unsworth, M.J., Chiang, C.W., Chen, C.S., Chen, C.C., Wu, F.T., Türkoğlu, E., Hsu, H.L., Hill, G.J., 2009. Magnetotelluric evidence for thick-skinned tectonics in central Taiwan. Geology 37, 711–714.

Bertrand, E.A., Unsworth, M.J., Chiang, C.W., Chen, C.S., Chen, C.C., Wu, F.T., Türkoğlu, E., Hsu, H.L., Hill, G.J., 2012. Magnetotelluric imaging beneath the Taiwan orogen: an arc-continent collision. Journal of Geophysical Research 117, B01402. http://dx.doi.org/10.1029/2011JB008688.

Chen, C.C., Chen, C.S., 1998. Preliminary result of magnetotelluric soundings in the fold-thrust belt of Taiwan and possible detection of dehydration. Tectonophysics 292, 101–117.

Chiang, C.W., Chen, C.C., Unsworth, M.J., Bertrand, E.A., Chen, C.S., Kieu, T.D., Hsu, H.L., 2010. The deep electrical structure of southern Taiwan and its tectonic implications. Terrestrial Atmospheric and Oceanic Sciences 21, 879–895.

Colangelo, G., Lapenna, V., Vallianatos, F., Nomikos, C., 2000. Investigating the time dynamics of geoelectrical signals measured in two seismotectonic environments of Mediterranean region: the Southern Apennine chain (S. Italy) and the Hellenic Arc (Crete island, Greece). Annali di Geofisica (43/2), 391–408.

Devroye, L., 1987. A Course on Density Estimation. Birkhauser, Boston.

Egbert, G.D., Eisel, M., Boyd, O.S., Morrison, H.F., 2000. DC trains and Pc3s: source effects in mid-latitude geomagnetic transfer functions. Geophysical Research Letters 124, 25–28. http://dx.doi.org/10.1029/1999GL008369.

Eisel, M., Egbert, G.D., 2001. On the stability of magnetotelluric transfer function estimates and reliability of their variances. Geophysical Journal International 144, 65–82. http://dx.doi.org/10.1046/j.1365-246x.2001.00292.

Esquivel, R.O., Angulo, J.C., Antolin, J., Dehesa, J.S., Lopez-Rosa, S., Flores-Gallegos, N., 2010. Analysis of complexity measures and information planes of selected molecules in position and momentum spaces. Physical Chemistry Chemical Physics 12, 7108–7116.

Fisher, R.A., 1925. Theory of statistical estimation. Proceedings of the Cambridge Philosophical Society 22, 700–725.

Frieden, B.R., 1990. Fisher information, disorder, and the equilibrium distributions of physics. Physical Review A 41, 4265–4276.

Hadjioannou, D., Vallianatos, F., Eftaxias, K., Hadjicontis, V., Nomikos, K., 1993. Subtraction of the telluric inductive component from the V.A.N measurements. Tectonophysics 224, 113–124.

Janicki, A., Weron, A., 1994. Simulation and chaotic behavior of stable stochastic processes. Marcel Dekker, New York.

Kappler, K.N., Morrison, H.F., Egbert, G.D., 2010. Long-term monitoring of ULF electromagnetic fields at Parkfield, California. Journal of Geophysical Research 115, B04406. http://dx.doi.org/10.1029/2009JB006421.

Li, D., Huang, Q., Chen, X., 2009. Error effects on magnetotelluric inversion. Chinese Journal of Geophysics 52, 268–274.

Lovallo, M., Telesca, L., 2011. Complexity measures and information planes of X-ray astrophysical sources. Journal of Statistical Mechanics, P03029.

Martin, M.T., Pennini, F., Plastino, A., 1999. Fisher’s information and the analysis of complex signals. Physics Letters A 256, 173–180.

Martin, M.T., Perez, J., Plastino, A., 2001. Fisher information and nonlinear dynamics. Physica A: Statistical Mechanics and its Applications 291, 523–532.

Nomikos, K., Vallianatos, F., 1997. Transient electric variations associated with large intermediate-depth earthquakes in South Aegean. Tectonophysics 269, 171–177.

Raykar, V.C., Duraiswami, R., 2006. Fast optimal bandwidth selection for kernel density estimation. In: Proceedings of the sixth SIAM International Conference on Data Mining, Bethesda, April 2006, pp. 524–528

Shannon, C.E., 1948. A mathematical theory of communication. Bell System Technical Journal 27 (379–423), 623–656.

Telesca, L., Lovallo, M., 2011. Analysis of the time dynamics in wind records by means of multifractal detrended fluctuation analysis and the Fisher–Shannon information plane. Journal of Statistical Mechanics, P07001.

Telesca, L., Lapenna, V., Vallianatos, F., Makris, J., Saltas, V., 2004. Multifractal features in short-term time dynamics of ULF geomagnetic field measured in Crete. Greece, Chaos Solitons and Fractals 21, 273–282.

Telesca, L., Lovallo, M., Ramirez-Rojas, A., Angulo-Brown, F., 2009. A nonlinear strategy to reveal seismic precursory signatures in earthquake-related self-potential signals. Physica A: Statistical Mechanics and its Applications 388, 2036–2040.

Telesca, L., Lovallo, M., Carniel, R., 2010. Time-dependent fisher information measure of volcanic tremor before 5 April 2003 paroxysm at Stromboli volcano, Italy. Journal of Volcanology and Geothermal Research 195, 78–82.

Telesca, L., Lovallo, M., Hsu, H.-L., Chen, C.-C., 2011. Analysis of dynamics in magnetotelluric data by using the Fisher–Shannon method. Physica A: Statistical Mechanics and its Applications 390, 1350–1355.

Troudi, M., Alimi, A.M., Saoudi, S., 2008. Analytical plug-in method for kernel density estimator applied to genetic neutrality study. EURASIP Journal on Advances in Signal Processing 2008, 8 (Article ID 739082).

Tzanis, A., Vallianatos, F., Gruszow, S., 2000. Identification and discrimination of transient electric earthquake precursors: fact, fiction and some possibilities. Physics of Earth and Planetary Interiors 121, 223–248.

Vallianatos, F., 1996. Magnetotelluric response of a random layered Earth. Geophysical Journal International 125, 557–583.

Vallianatos, F., 2002. A note on the topoographic distortion of the Magnetotelluric impedance. Annali di Geofisica 45 (2), 313–321.

Vallianatos, F., Eftaxias, K., 1993. A model for the influence of the local inhomogeneities on the magnetotelluric measurements at V.A.N stations in Greece. Tectonophysics 224, 125–130.